

Cost Benefit Analysis of Three Non-Identical Machine Model with Priority in Operation and Repair

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Received: 15/Aug/2020

Revised: 10/May/2021

Accepted: 07/June/2021

Abstract

The paper proposes a new real life model and the main aim is to examine the cost benefit analysis of Textile Industry model subject to different failure and repair strategies. The reliability model comprises of three units i.e Spinning machine (S), Weaving machine (W), Colouring and Finishing machine(Cf). The working principal of the model starts with spinning machine (S) where in unit S is in operative state while as weaving machine, Colouring and Finishing machine are in ideal state. Complete failure of system is observed when all three units of system i.e. S,W and Cf are in down state. Repairperson is always available to carry out the repair activities in the system in which first priority in repair is given to Colouring and Finishing machine followed by Spinning and weaving machine. The proposed model attempts to maximize the reliability of a real life system. Reliability measures such as Mean Sojourn time, Mean time to system failure, Profit analysis of system are examined to define the performance of the reliability characteristics. For concluding the study of such model, different stochastic measures are analyzed in steady state using regenerative point technique. The tables are prepared for arbitrary values of the parameters to show the performance of some important reliability measures and to check the efficiency of the model under such situations.

Keywords: Reliability Measures; Mean Sojourn Time; Laplace Transformation; Laplace -Stieltjes transformation.

1- Introduction

In the present scenario of global viable market, the basic objectives of industries is to reduce the production cost and enhance the productivity of manufacturing systems of processing Industries. To meet the increasing demand of the society, the industries must offer long term production. The efficiency and accessibility of industry can be made profitable in terms of higher production and lower maintenance costs. Availability of complex systems can be improved by considering different techniques of reliability like maintenance, inspection, repairs, priority to repair of the failed unit and so on. In addition, these techniques in industrial design can substantially strengthen operational dependability through better system design. A very trivial change in the technique of reliability can cause a profound change in the operational dependability, that is why it is

vital to specify model reliability targets before any strategy plan is taken into consideration .

There are systems where upon failure in a complex system the repair of a component is given preference over the other component depending upon the significance of the component. Such systems are called priority systems. When a unit of lower priority is under repair and a unit of higher priority arrives for repair, the lower priority unit is pre-empted and higher priority unit is repaired. After completion of the service of higher priority unit, the lower higher priority unit is given service. In reliability modeling, it is found that accessibility and performance of failed and repairable systems can be enriched using redundancy technique. But due to the high cost, it is not always possible to keep the unit as spare. Also, it sometimes becomes essential to prioritize one unit over the other to increase performance, availability and hence the efficiency of the system. The idea of priority in repair discipline in reliability modeling has been

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considered by various authors researchers in order to make the system available for required service with least possible operational cost.

Kumar et al. [2] obtained reliability analysis of a warm standby model performing in normal/abnormal environment under certain assumptions where the distribution of failure time has been taken as negative exponential and the repair, inspection and replacement time distribution has been taken as arbitrary. Chander [11] discussed the two non-identical reliability models in which the failure and repair time of each unit were independent and uncorrelated random variables. The author obtained the various reliability measures using Regenerative Point Technique (R.P.T). Also, Zhang Wang [15] studies a cold standby system with priority in use and repair. Malik et al. [12] studied cost- benefit analysis of a model under and gave priority to repair and degradation. The numerical and graphical behaviour of important reliability measures has been obtained for arbitrary values of costs and parameter. Further, Kadyan et al. [6] considered reliability performance of model under priority to repair and degradation. The authors obtained the availability and profit of the system using Regenerative point technique (RPT). Sureria and Malik [5], Malik [13] analyzed computer system with priority to software replacement over hardware replacement and obtained the reliability and economic measures of computer system under certain assumptions. The model has been analyzed stochastically by using R.P.T and the results of the model has been compared with the existing model. Chib et al. [3] assumed two types of repair facilities for stochastic analysis of two unit priority system. Rathee and Chander [9] carried out the reliability analysis of parallel system in which he gave priority to repair over maintenance. Munday and Malik [14] obtained profit analysis of a Computer System with Priority for hardware redundancy and repair. Recently Kumar et al. [4] studied profit analysis of computer systems with priority and maximum operation time. The author gave priority to software upgradation over hardware repair and replacement and hence obtained the profit of the system. The profit of the system has been compared with some existing model in literature. Also Bashir et al. [7] used the technique of inspection of degraded unit and obtained cost analysis of two unit model. Gorjian Jolfaei [8] carried out reliability analysis of power generation engines in a wastewater treatment plant. Kumar and Malik [1] examines the weathering server system with identical element in warm standby and used the concept of first come first serve repair policy for redundant systems in weather conditions. The various reliability measures of the system has been obtained using R.P.T. The author proved that the concept of first come first serve repair policy proved to be economically beneficial. Sharma and joorel [10] developed a model for fruit juice manufacturing system for its reliability analysis,

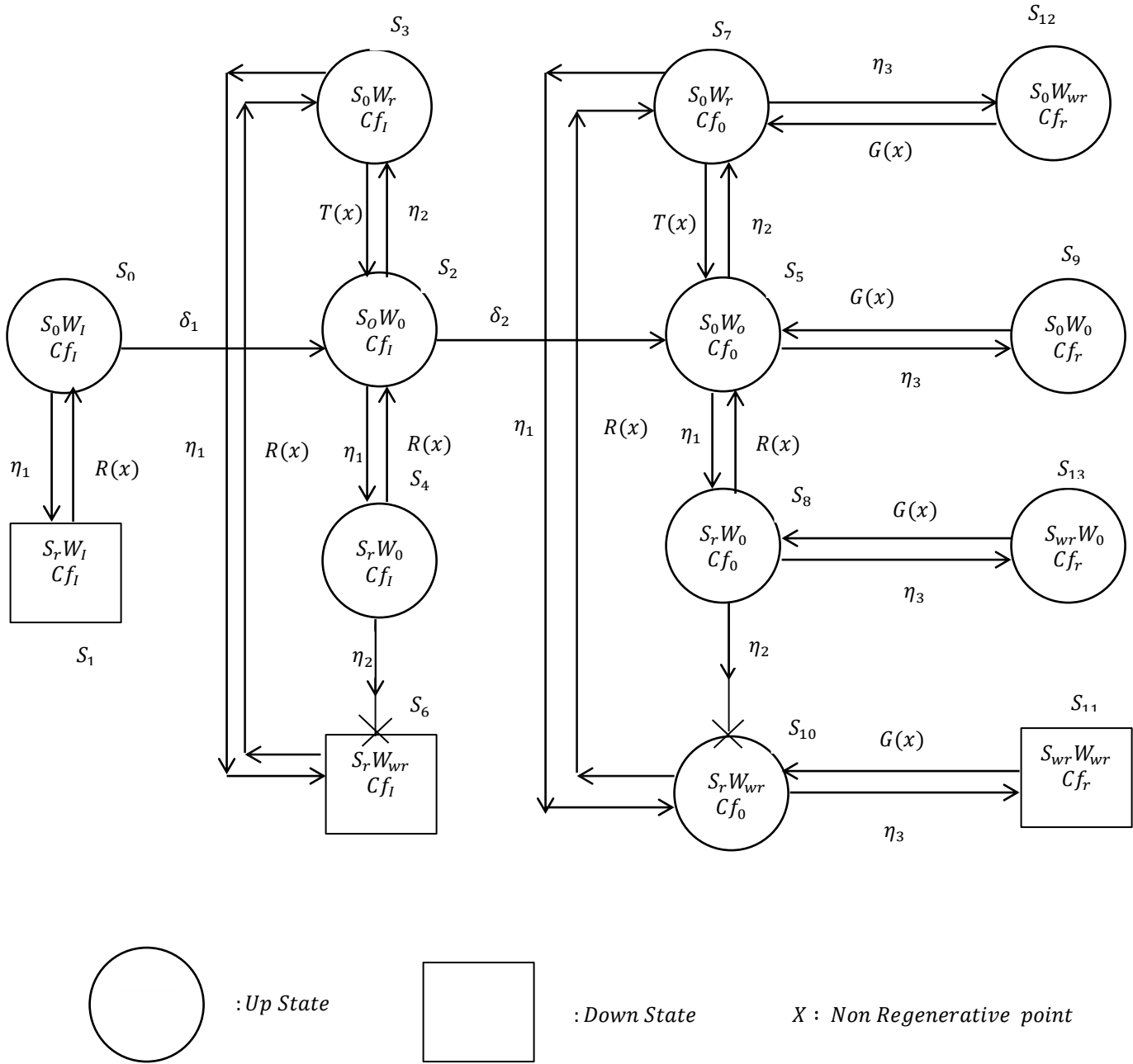
which consists of four non identical units viz. Fruit cutter, Fruit pulper, Sterlizer and Homogenizer. The author used the concept of emergency repair in which the team of experts is available on the failure of Sterlizer or Homogenizer, to carry out the failure activities. [16] Obtained the reliability of two unit cold standby model under the assumption that the system is operating under different weather conditions and gave priority to preventive maintenance over inspection. To determine the nature of the system, various system effectiveness has been obtained using regenerative point technique and semi markov process. [17] Analysed the concrete mixture plant which comprises of seven units subject to the coverage factor by considering profust reliability approach which is based on probability assumption as well as fuzzy state assumption. The performance analysis of the system has been done using Markov birth–death model and differential equations solved by Runge–Kutta method. The results of the proposed model have proved beneficial for system designers. [18] have made an effort to obtain the system performance of sewage treatment plant by using markovian birth-death process. Various system measures like reliability, availability, maintainability and obtainability of the system has been obtained. [19] explored the generators in steam turbine power plant and obtained the availability and profit of generators. Using supplementary variable technique amd markov birth-death process different system measures has been evaluated. Results of the system have been obtained for availability and profit that would be useful for system designers to enrich the reliability and thus performance of plant

The basic aim of the paper is to obtain the profit evaluation of textile industry with exponential failure time distribution subject to different repair strategies. The System consists of three non-identical components Spinning (S), Weaving (W), Coloring and Finishing (Cf). Initially unit S is in working mode whereas unit W and Cf are in idle state. Failure of the system occurs only if all the unit fails. Single server is available in the system to carry out repair activities and Cf gets first priority over repair followed by S and W. The failure time (FT) \sim exp. (η). The repair time distribution is taken as general.

2- Notations and Symbols

- η_1, η_2, η_3 = failure rates of unit S, W, Cf respectively.
- δ_1, δ_2 = activation rates of unit W and Cf from idle to operation respectively
- $R(t), T(t), G(t)$ = cdf of repair time of the failed unit S, W, Cf respectively
- * = Laplace transform i.e. $f^*(s) = \int_0^{\infty} e^{-st}f(t)dt$

TRANSITION DIAGRAM



\sim = Laplace-Stieltjes transform i.e. $\tilde{F}(s) = \int_0^\infty e^{-st} dF(t)$.

- $S_0, W_0, Cf_0 = S, W, Cf$ are normal and operative.
- $S_r, W_r, Cf_r = S, W, Cf$ are under repair.
- $S_{wr}, W_{wr}, Cf_{wr} = S, W, Cf$ are waiting for repair.
- $W_l, Cf_l = W, Cf$ is in ideal state.
- S_0 = Total possible states of the system, $i = 0, 1, 2, 3 \dots 13$

- $S_0 = [S_0, W_l, Cf_l]$
- $S_1 = [S_r, W_l, Cf_l]$
- $S_2 = [S_0, W_0, Cf_l]$
- $S_3 = [S_0, W_r, Cf_l]$
- $S_4 = [S_r, W_0, Cf_l]$
- $S_5 = [S_0, W_0, Cf_0]$
- $S_6 = [S_r, W_{wr}, Cf_l]$
- $S_7 = [S_0, W_r, Cf_0]$
- $S_8 = [S_r, W_0, Cf_0]$
- $S_9 = [S_0, W_0, Cf_r]$
- $S_{10} = [S_r, W_{wr}, Cf_0]$
- $S_{11} = [S_{wr}, W_{wr}, Cf_r]$
- $S_{12} = [S_0, W_{wr}, Cf_r]$
- $S_{13} = [S_{wr}, W_0, Cf_r]$

3- Transition Probabilities

Let $Q_{ij}(t)$ represents the transition probability from state i to j in transient state. Then the various transition probabilities, can be written as follows:

$$Q_{01}(t) = \eta_1 \int_0^t e^{-(\eta_1 + \delta_1)u} du$$

$$Q_{02}(t) = \delta_1 \int_0^t e^{-(\eta_1 + \delta_1)u} du$$

$$Q_{10}(t) = R(u) du$$

$$Q_{23}(t) = \eta_2 \int_0^t e^{-(\eta_1 + \eta_2 + \delta_2)u} du$$

$$Q_{24}(t) = \eta_1 \int_0^t e^{-(\eta_1 + \eta_2 + \delta_2)u} du$$

$$Q_{25}(t) = \delta_2 \int_0^t e^{-(\eta_1 + \eta_2 + \delta_2)u} du$$

$$Q_{32}(t) = \int_0^t e^{-(\eta_1)u} dtu$$

$$Q_{36}(t) = \eta_1 \int_0^t e^{-(\eta_1)u} \bar{T}(u) du$$

$$Q_{42}(t) = \int_0^t e^{-(\eta_2)u} dr(u)$$

$$Q_{57}(t) = \eta_2 \int_0^t e^{-(\eta_1 + \eta_2 + \eta_3)u} du$$

$$Q_{58}(t) = \eta_1 \int_0^t e^{-(\eta_1 + \eta_2 + \eta_3)u} du$$

$$Q_{59}(t) = \eta_3 \int_0^t e^{-(\eta_1 + \eta_2 + \eta_3)u} du$$

$$Q_{63}(t) = R(u) du$$

$$Q_{75}(t) = \int_0^t e^{-(\eta_2 + \eta_3)u} du$$

$$Q_{7,10}(t) = \eta_1 \int_0^t e^{-(\eta_1 + \eta_3)u} \bar{T}(u) du$$

$$Q_{7,12}(t) = \eta_3 \int_0^t e^{-(\eta_1 + \eta_3)u} \bar{T}(u) du$$

$$Q_{8,13}(t) = \eta_3 \int_0^t e^{-(\eta_2 + \eta_3)u} \bar{R}(u) du$$

$$Q_{8,11}(t) = \eta_2 \eta_3 \int_0^t \int_0^v e^{-(\eta_2)u} du e^{-(\eta_3)v} \bar{R}(v) dv$$

$$Q_{85}(t) = \int_0^t e^{-(\eta_2 + \eta_3)u} dR(u)$$

$$Q_{87}(t) = \eta_2 \int_0^t \int_0^v e^{-(\eta_2)u} du e^{-(\eta_3)v} dR(v) dv$$

$$Q_{95}(t) = \int_0^t G(u) du$$

$$Q_{10,7}(t) = \int_0^t e^{-(\eta_3)u} dR(u)$$

$$Q_{10,11}(t) = \int_0^t e^{-(\eta_3)u} \bar{R}(u) du$$

$$Q_{11,10}(t) = Q_{12,7}(t) = Q_{13,8}(t) = \int_0^t G(u) du$$

In transition probabilities by taking $t \rightarrow \infty$ we get the steady state probabilities i.e $p_{ij} = Q_{ij}(\infty) = \int_0^\infty Q_{ij}(t) dt$ and

it can be easily verified that $\sum_j p_{ij} = 1$ for all possible values of i , i.e

$$p_{01} + p_{02} = 1$$

$$p_{10} = 1$$

$$p_{23} + p_{24} + p_{25} = 1$$

$$p_{32} + p_{36} = 1$$

$$p_{42} + p_{43}^{(6)} = 1$$

$$p_{57} + p_{58} + p_{59} = 1$$

$$p_{63} = 1$$

$$p_{75} + p_{7,10} + p_{7,12} = 1$$

$$p_{85} + p_{87}^{(10)} + p_{87}^{(11)} + p_{8,13} = 1$$

$$p_{95} = 1$$

$$p_{10,7} + p_{10,11} = 1$$

$$p_{11,10} = p_{12,7} = p_{13,8} = 1$$

3.1 Mean Sojourn Times(MST)

The MST ϕ_i is the expected time taken by a system in a given state before transiting to any other state. If T_i is the sojourn time of S_i then MST ϕ_i is given by:

$$\phi_i = \int_0^{\infty} P(T_i > t) dt$$

$$\phi_0 = \int_0^{\infty} e^{-(\eta_1 + \delta_1)t} dt = \frac{1}{\eta_1 + \delta_1}$$

$$\phi_1 = \int_0^{\infty} \bar{R}(t) dt$$

$$\phi_2 = \int_0^{\infty} e^{-(\eta_1 + \eta_2 + \delta_2)t} dt = \frac{1}{\eta_1 + \eta_2 + \delta_2}$$

$$\phi_3 = \int_0^{\infty} e^{-(\eta_1)t} \bar{T}(t) dt = \frac{1}{\eta_1} [1 - \tilde{T}(\eta_1)]$$

$$\phi_4 = \int_0^{\infty} e^{-(\eta_2)t} \bar{R}(t) dt = \frac{1}{\eta_2} [1 - \tilde{R}(\eta_2)]$$

$$\phi_5 = \int_0^{\infty} e^{-(\eta_1 + \eta_2 + \eta_3)t} dt$$

$$\phi_6 = \int_0^{\infty} \bar{R}(t) dt$$

$$\phi_7 = \int_0^{\infty} e^{-(\eta_2 + \eta_3)t} \bar{T}(t) dt = \frac{1}{\eta_2 + \eta_3} [1 - \tilde{T}(\eta_2 + \eta_3)]$$

$$\phi_8 = \int_0^{\infty} e^{-(\eta_2 + \eta_3)t} \bar{R}(t) dt = \frac{1}{\eta_2 + \eta_3} [1 - \tilde{R}(\eta_2 + \eta_3)]$$

$$\phi_9 = \phi_{11} = \phi_{12} = \phi_{13} = \int_0^{\infty} \bar{G}(t) dt$$

$$\phi_{10} = \int_0^{\infty} e^{-(\eta_3)t} \bar{R}(t) dt = \frac{1}{\eta_3} [1 - \tilde{R}(\eta_3)]$$

4- Mean Time to System Failure

We define T_i as the time to system failure and $\pi_i(t)$ be the c.d.f. of the time to system failure for the first time when the system starts operation from state S_i . We find the equations of $\pi_i(t)$ for various values of i , by using regenerative point technique.

Therefore, MTSF for different states is given by:

$$\pi_0(t) = Q_{01}(t) + Q_{02}(t) \otimes \pi_2(t)$$

$$\pi_2(t) = Q_{23}(t) \otimes \pi_3(t) + Q_{24}(t) \otimes \pi_4(t) + Q_{25}(t) \otimes \pi_5(t)$$

$$\pi_3(t) = Q_{32}(t) \otimes \pi_2(t) + Q_{36}(t)$$

$$\pi_4(t) = Q_{42}(t) \otimes \pi_2(t) + Q_{46}(t)$$

$$\pi_5(t) = Q_{57}(t) \otimes \pi_7(t) + Q_{58}(t) \otimes \pi_8(t) + Q_{59}(t) \otimes \pi_9(t)$$

$$\pi_7(t) = Q_{75}(t) \otimes \pi_5(t) + Q_{7,10}(t) \otimes \pi_{10}(t)$$

$$+ Q_{7,12}(t) \otimes \pi_{12}(t)$$

$$\pi_8(t) = Q_{85}(t) \otimes \pi_5(t) + Q_{8,10}(t) \otimes \pi_{10}(t) +$$

$$Q_{8,13}(t) \otimes \pi_{13}(t)$$

$$\pi_9(t) = Q_{95}(t) \otimes \pi_5(t)$$

$$\pi_{10}(t) = Q_{10,7}(t) \otimes \pi_7(t) + Q_{10,11}(t)$$

$$\pi_{12}(t) = Q_{12,7}(t) \otimes \pi_7(t)$$

$$\pi_{13}(t) = Q_{13,8}(t) \otimes \pi_8(t)$$

Taking L.S.T of above equations, and simplifying for $\tilde{\pi}_0(s)$,

$$\tilde{\pi}_0(s) = \frac{N_1(s)}{D_1(s)}$$

On taking $s \rightarrow 0$ in above equation and $\tilde{Q}_{ij}(s) \rightarrow p_{ij}$, as $s \rightarrow 0$, we have

$$\tilde{\pi}_0(s) = \frac{N_1(s)}{D_1(s)} = 1 \text{ implies that } \pi_0(t) \text{ is a proper cdf.}$$

Therefore, MTSF is given by

$$E(T) = -\frac{d\tilde{\pi}_0(s)}{ds}\Big|_{s=0} = \frac{D'_1(0) - N'_1(0)}{D_1(0)}$$

Where,

$$D'_1(0) - N'_1(0) = \left[\phi_0 \{1 - p_{23}p_{32} - p_{24}p_{42}\} + \phi_2 p_{02} + \phi_3 p_{23} p_{02} + \phi_4 p_{02} p_{24} \right] + \left[(1 - p_{7,10} p_{10,7} - p_{7,12}) \right] \left[(1 - p_{8,13}) - \{p_{58} p_{85} + p_{59} (1 - p_{8,13})\} \right] - \left[p_{75} (p_{57} (1 - p_{8,13}) + p_{58} p_{8,10} p_{10,7}) \right] + \phi_5 p_{75} (1 - p_{8,13}) (1 - p_{23} p_{32} - p_{24} p_{42}) + \phi_7 \{p_{57} (1 - p_{8,13}) + p_{58} p_{8,10} p_{10,7}\} (1 - p_{23} p_{32} - p_{24} p_{42}) + \phi_8 p_{58} (1 - p_{7,10} p_{10,7} - p_{7,12}) (1 - p_{23} p_{32} - p_{24} p_{42}) + \phi_9 p_{59} (1 - p_{8,13}) (1 - p_{7,10} p_{10,7} - p_{7,12}) (1 - p_{23} p_{32} - p_{24} p_{42}) + \phi_{10} \left[p_{7,10} \left[(1 - p_{8,13}) - \{p_{58} p_{85} + p_{59} (1 - p_{8,13})\} \right] + p_{58} p_{8,10} \right] (1 - p_{23} p_{32} - p_{24} p_{42}) + \phi_{12} p_{7,12} \left[(1 - p_{8,13}) - \{p_{58} p_{85} + p_{59} (1 - p_{8,13})\} \right] (1 - p_{23} p_{32} - p_{24} p_{42}) + \phi_{13} \left[(p_{8,13} - p_{59} p_{8,13}) (1 - p_{7,10} p_{10,7} - p_{7,12}) - p_{57} p_{75} p_{8,13} \right] (1 - p_{23} p_{32} - p_{24} p_{42})$$

$$D_1(0) = (1 - p_{23} p_{32} - p_{24} p_{42}) \left[\{1 - p_{7,10} p_{10,7} - p_{7,12} p_{12,7}\} \left\{ (1 - p_{8,13} p_{13,8}) - \{p_{58} p_{85} + p_{59} (1 - p_{8,13})\} \right\} - \{p_{75} (p_{57} (1 - p_{8,13}) + p_{58} p_{8,10} p_{10,7})\} \right]$$

5- Availability Analysis

We define availability, $A_i(t)$ as the probability that the system is in upstate and is available to serve its required purpose when it initially start from regenerative state S_i

The availability equations for different states is given as:

$$\begin{aligned} A_0(t) &= M_0(t) + q_{01}(t) \odot A_1(t) + q_{02}(t) \odot A_2(t) \\ A_1(t) &= q_{10}(t) \odot A_0(t) \\ A_2(t) &= M_2(t) + q_{23}(t) \odot A_3(t) + q_{24}(t) \odot A_4(t) + q_{24}(t) \odot A_4(t) \\ A_3(t) &= M_3(t) + q_{32}(t) \odot A_2(t) + q_{36}(t) \odot A_6(t) \\ A_4(t) &= M_4(t) + q_{42}(t) \odot A_2(t) + q_{43}^{(6)}(t) \odot A_3(t) \\ A_5(t) &= M_0(t) + q_{01}(t) \odot A_1(t) + q_{02}(t) \odot A_2(t) \\ A_6(t) &= q_{63}(t) \odot A_3(t) \\ A_7(t) &= M_7(t) + q_{75}(t) \odot A_5(t) + q_{7,10}(t) \odot A_{10}(t) + q_{7,12}(t) \odot A_{12}(t) \\ A_8(t) &= M_8(t) + q_{85}(t) \odot A_5(t) + q_{87}^{(10)}(t) \odot A_7(t) + q_{8,11}^{(10)}(t) \odot A_{11}(t) + q_{8,13}(t) \odot A_{13}(t) \\ A_9(t) &= M_9(t) + q_{95}(t) \odot A_5(t) \\ A_{10}(t) &= M_{10}(t) + q_{10,7}(t) \odot A_7(t) + q_{10,11}(t) \odot A_{11}(t) \end{aligned}$$

$$\begin{aligned} A_{11}(t) &= q_{11,10}(t) \odot A_{10}(t) \\ A_{12}(t) &= M_{12}(t) + q_{12,7}(t) \odot A_7(t) \\ A_{13}(t) &= M_{13}(t) + q_{13,8}(t) \odot A_8(t) \end{aligned}$$

Taking the LT of above equations, we get the set of linear equations in $A_i^*(s)$ and solving for $A_0^*(s)$, we get

$$A_0^*(s) = \frac{N_2(s)}{D_2(s)}$$

The steady state availability is given as:

$$A_0 = \lim_{s \rightarrow 0} s A_0^*(s) = N_2(0)/D_2'(0)$$

where,

$$N_2(0) = \left[\{p_{32} (1 - p_{23}) - p_{24} p_{32}\} \phi_0 + \{p_{02} ((\phi_2 + p_{24} \phi_4) p_{32} + (p_{23} p_{24} p_{43}))\} \right] \left[\{p_{10,7} - p_{7,10} p_{7,12}\} \left\{ (1 - p_{8,13}) (1 - p_{59}) - p_{58} p_{85} \right\} - \{p_{57} (1 - p_{8,13}) + p_{58} p_{87}^{(10)}\} (p_{75} p_{10,7}) \right] + p_{02} p_{25} \left[\{(\phi_5 + p_{59} \phi_9) (1 - p_{8,13}) + p_{58} (\phi_8 + p_{8,13} \phi_{13})\} (p_{10,7} (1 - p_{7,12})) + \{p_{57} (1 - p_{8,13}) + p_{58} p_{87}^{(10)}\} \left\{ (\phi_7 + p_{7,12} \phi_{12}) (1 - p_{7,10}) p_{10,7} + p_{7,10} \{ \phi_{10} + p_{10,7} (\phi_7 + p_{7,12} \phi_{12}) \} \right\} \right] \quad (1)$$

$$\begin{aligned} D_2'(0) &= [\phi_5 (1 - p_{8,13}) + \phi_8 p_{58}] p_{02} p_{25} p_{32} p_{75} p_{10,7} + [\phi_7 \{p_{57} (1 - p_{8,13}) + p_{58} p_{87}^{(10)}\} + \phi_9 p_{59} p_{75} (1 - p_{8,13})] p_{02} p_{25} p_{32} p_{10,7} + \phi_{10} \left[(1 - p_{7,12}) \left\{ (1 - p_{8,13}) (1 - p_{59}) - p_{58} p_{85} \right\} - p_{75} \{p_{57} (1 - p_{8,13}) + p_{58} p_{87}^{(10)}\} \right] p_{02} p_{32} p_{25} + \phi_{11} \left[p_{10,11} (1 - p_{7,12}) \left\{ (1 - p_{8,13}) (1 - p_{59}) - p_{58} p_{85} \right\} - p_{75} p_{10,11} \{p_{57} (1 - p_{8,13}) + p_{58} p_{87}^{(10)}\} \right] p_{02} p_{32} p_{25} + [\phi_{12} \{ (1 - p_{8,13}) (1 - p_{59}) - p_{58} p_{85} \} p_{7,12} + \phi_{13} p_{75} p_{8,13} p_{58}] p_{10,7} p_{02} p_{25} p_{32} \quad (2) \end{aligned}$$

6- Busy Period Analysis (BPA)

It is the probability that the repairman is occupied in repairing the failed component at time 't' given that the system has started from regenerative state S_i at $t = 0$. The BPA for different states is given by

$$\begin{aligned} B_0(t) &= q_{01}(t) \odot B_1(t) + q_{02}(t) \odot B_2 \\ B_1(t) &= M_1 + q_{10}(t) \odot B_0(t) \\ B_2(t) &= q_{23}(t) \odot B_3(t) + q_{24}(t) \odot B_4 + q_{25}(t) \odot B_5 \\ B_3(t) &= M_3 + q_{32}(t) \odot B_2(t) + q_{36}(t) \odot B_6(t) \\ B_4(t) &= M_4 + q_{42}(t) \odot B_2(t) + q_{43}^{(6)}(t) \odot B_3(t) \end{aligned}$$

$$B_5(t) = M_5 + q_{57}(t) \odot B_7(t) + q_{58}(t) \odot B_8(t) + q_{59}(t) \odot B_9(t)$$

$$B_6(t) = M_6 + q_{63}(t) \odot B_3(t)$$

$$B_7(t) = M_7 + q_{75}(t) \odot B_5(t) + q_{7,10}(t) \odot B_{10}(t) + q_{7,12}(t) \odot B_{12}$$

$$B_8(t) = M_8 + q_{85}(t) \odot B_5(t) + q_{87}^{(10)}(t) \odot B_{10}(t) + q_{8,11}^{(10)}(t) \odot B_{11}(t) + q_{8,13}(t) \odot B_{13}(t)$$

$$B_9(t) = M_9 + q_{95}(t) \odot B_5(t)$$

$$B_{10}(t) = M_{10} + q_{10,7}(t) \odot B_7(t) + q_{10,11}(t) \odot B_{11}(t)$$

$$B_{11}(t) = M_{11} + q_{11,10}(t) \odot B_{10}(t)$$

$$B_{12}(t) = M_{12} + q_{12,7}(t) \odot B_7(t)$$

$$B_{13}(t) = M_{13} + q_{13,8}(t) \odot B_8(t)$$

Taking the LT of above equations and solving for $B_0^*(s)$,

$$B_0^*(s) = \frac{N_3(s)}{D_3(s)}$$

In steady state, BPA is given by

$$B_0 = \lim_{s \rightarrow 0} s B_0^*(s) \frac{N_3(0)}{D_3'(0)}$$

$$N_3(0) = \phi_8 p_{58} p_{02} p_{25} p_{32} p_{75} p_{10,7} + \left[\phi_7 \left\{ p_{57} (1 - p_{8,13}) + p_{58} p_{87}^{(10)} \right\} \right] p_{02} p_{25} p_{32} p_{10,7} + \left[\phi_{10} (1 - p_{7,12}) \left\{ (1 - p_{8,13}) (1 - p_{59}) - p_{58} p_{85} \right\} - p_{75} \left\{ p_{57} (1 - p_{8,13}) + p_{58} p_{87}^{(10)} \right\} \right] p_{02} p_{32} p_{25} + \phi_{11} \left[p_{10,11} (1 - p_{7,12}) \left\{ (1 - p_{8,13}) (1 - p_{59}) - p_{58} p_{85} \right\} p_{75} p_{10,11} \left\{ p_{57} (1 - p_{8,13}) + p_{58} p_{87}^{(10)} \right\} \right] p_{02} p_{32} p_{25} + \left[\phi_{12} \left\{ (1 - p_{8,13}) (1 - p_{59}) - p_{58} p_{85} \right\} p_{7,12} + \phi_{13} p_{75} p_{8,13} p_{58} \right] p_{02} p_{25} p_{32} p_{10,7}$$

And $D_3'(0) = D_2'(0)$ given by (2)

7- Expected No. of Visits by Repairman

We denote $V_i(t)$ as the expected no. of visits by the repairperson to repair the failed unit. The expected no of visits for different states is given as:

$$V_0(t) = q_{01}(t) \odot V_1(t) + q_{02}(t) \odot V_2(t)$$

$$V_1(t) = q_{10}(t) \odot V_0(t)$$

$$V_2(t) = q_{23}(t) \odot V_3(t) + q_{24}(t) [1 + V_4(t)] + q_{25}(t) \odot V_5(t)$$

$$V_3(t) = q_{32}(t) \odot V_2(t) + q_{36}(t) \odot V_6(t)$$

$$V_4(t) = q_{42}(t) \odot V_2(t) + q_{43}(t) \odot V_3(t)$$

$$V_5(t) = q_{57}(t) [1 + V_7(t)] + q_{58}(t) [1 + V_8(t)] + q_{59}(t) [1 + V_9(t)]$$

$$V_6(t) = q_{63}(t) \odot V_3(t)$$

$$V_7(t) = q_{75}(t) \odot V_5(t) + q_{7,10}(t) \odot V_{10}(t) + q_{7,12}(t) \odot V_{12}(t)$$

$$V_8(t) = q_{85}(t) \odot V_5(t) + q_{8,13}(t) \odot V_{13}(t) + q_{87}^{(10)}(t) \odot V_7(t) + q_{8,11}^{(10)}(t) \odot V_{11}(t)$$

$$V_9(t) = q_{95}(t) \odot V_5(t)$$

$$V_{10}(t) = q_{10,7}(t) \odot V_7(t) + q_{10,11}(t) \odot V_{11}(t)$$

$$V_{11}(t) = q_{11,10}(t) \odot V_{10}(t)$$

$$V_{12}(t) = q_{12,7}(t) \odot V_7(t)$$

$$V_{13}(t) = q_{13,8}(t) \odot V_8(t)$$

Taking the LT of above equations and solving for $V_0^*(s)$,

$$V_0^*(s) = \frac{N_4(s)}{D_4(s)}$$

In steady state, the no. of visits per unit time is given by

$$V_0(0) = \lim_{s \rightarrow 0} s V_0^*(s) = N_4(0) / D_4'(0)$$

Where,

$$N_4(0) = \left[p_{01} \left\{ (1 - p_{24} p_{42}) p_{32} - \left\{ p_{23} + p_{24} p_{43}^{(6)} \right\} p_{32} \right\} + p_{02} (p_{23} + p_{24} + p_{25}) p_{32} \right] \left[p_{10,7} p_{7,5} (1 - p_{85} p_{58} - p_{59} (1 - p_{8,13})) \right] - \left\{ p_{57} (1 - p_{8,13}) + p_{58} p_{87}^{(10)} \right\} p_{75} p_{10,11} + \left\{ p_{02} p_{25} p_{32} (p_{57} + p_{58} + p_{59}) (1 - p_{8,13}) \right\} p_{10,7} p_{75}$$

And $D_4'(0) = D_2'(0)$ given in (2)

8- Profit Analysis

The profit encountered to the model in steady state by considering different costs and revenues, is calculated as:

$$P_1 = K_0 A_0 - K_1 B_0 - K_2 V_0$$

Where

K_0 = Revenue per unit up time of the system,

K_1 = Cost per unit time for which the repair is busy,

K_2 = Cost per unit visits by the repairman.

9- Behavior of Reliability Measures:

In this section, for practical illustration, the behavior of some important reliability measures has been examined for some specific values of parameters.

Table 1. Performance of η_1 and fixed parameters $\eta_2, \eta_3, \delta_1, \delta_2, \alpha_1, \alpha_2$ and α_3 on MTSF.

η_1	MTSF		
	$\eta_2 = 0.10$ $\eta_3 = 0.04$ $\delta_1 = 0.74$ $\delta_2 = 0.60$ $\alpha_1 = 0.47$ $\alpha_2 = 0.24$ $\alpha_3 = 0.33$	$\eta_2 = 0.08$ $\eta_3 = 0.02$ $\delta_1 = 0.79$ $\delta_2 = 0.69$ $\alpha_1 = 0.34$ $\alpha_2 = 0.39$ $\alpha_3 = 0.37$	$\eta_2 = 0.15$ $\eta_3 = 0.06$ $\delta_1 = 0.64$ $\delta_2 = 0.50$ $\alpha_1 = 0.37$ $\alpha_2 = 0.34$ $\alpha_3 = 0.39$
0.1	3.9904	5.2570	2.8029
0.2	2.9559	3.9690	2.3256
0.3	2.3585	3.2605	2.0005
0.4	1.9677	2.8004	1.7637
0.5	1.6917	2.4719	1.5834
0.6	1.4863	2.2227	1.4416
0.7	1.3276	2.0256	1.3271
0.8	1.2013	1.8648	1.2328
0.9	1.0985	1.7307	1.1537
1.0	1.0065	1.6612	1.0653

Table 2. Performance of α_1 and fixed parameters $\eta_2, \eta_3, \delta_1, \delta_2, \eta_1, \alpha_2$ and α_3 on MTSF.

α_1	MTSF		
	$\eta_2 = 0.13$ $\eta_3 = 0.25$ $\delta_1 = 0.31$ $\delta_2 = 0.33$ $\eta_1 = 0.27$ $\alpha_2 = 0.03$ $\alpha_3 = 0.33$	$\eta_2 = 0.15$ $\eta_3 = 0.18$ $\delta_1 = 0.34$ $\delta_2 = 0.43$ $\eta_1 = 0.29$ $\alpha_2 = 0.04$ $\alpha_3 = 0.33$	$\eta_2 = 0.11$ $\eta_3 = 0.21$ $\delta_1 = 0.31$ $\delta_2 = 0.33$ $\eta_1 = 0.23$ $\alpha_2 = 0.02$ $\alpha_3 = 0.33$
0.1	1.8571	1.6701	2.1038
0.2	2.0716	1.8292	2.4664
0.3	2.1878	1.9227	2.6476
0.4	2.2646	1.9869	2.7667
0.5	2.3221	2.0350	2.8596
0.6	2.3680	2.0727	2.9397
0.7	2.4054	2.1019	3.0123
0.8	2.4351	2.1230	3.0792
0.9	2.4563	2.1352	3.1401
1.0	2.4677	2.1369	3.1938

Table 3. Performance of η_1 and fixed parameters $\eta_2, \eta_3, \delta_1, \delta_2, \alpha_1, \alpha_2, \alpha_3, k_0, k_1$ and k_2 on Profit.

η_1	Profit		
	$\eta_2 = 0.81$ $\eta_3 = 0.95$ $\delta_1 = 0.91$ $\delta_2 = 0.39$ $\alpha_1 = 0.56$ $\alpha_2 = 0.32$ $\alpha_3 = 0.33$ $k_0 = 1000$ $k_1 = 100$ $k_2 = 50$	$\eta_2 = 0.79$ $\eta_3 = 0.93$ $\delta_1 = 0.91$ $\delta_2 = 0.39$ $\alpha_1 = 0.55$ $\alpha_2 = 0.35$ $\alpha_3 = 0.33$ $k_0 = 1000$ $k_1 = 120$ $k_2 = 90$	$\eta_2 = 0.89$ $\eta_3 = 0.97$ $\delta_1 = 0.81$ $\delta_2 = 0.32$ $\alpha_1 = 0.54$ $\alpha_2 = 0.33$ $\alpha_3 = 0.33$ $k_0 = 950$ $k_1 = 120$ $k_2 = 90$
0.1	2.5482	2.4729	2.8455
0.2	2.1945	2.2126	2.4681
0.3	1.8936	2.0004	2.1495
0.4	1.6221	1.8157	1.8639
0.5	1.3656	1.6459	1.5954
0.6	1.1151	1.4828	1.3339
0.7	0.8643	1.3212	1.0724
0.8	0.6091	1.1574	0.8061
0.9	0.3463	0.9888	0.5318
1.0	0.0737	0.8134	0.2468

Table 4. Performance of α_1 and fixed parameters $\eta_2, \eta_3, \delta_1, \delta_2, \eta_1, \alpha_2, \alpha_3, k_0, k_1$ and k_2 on Profit.

α_1	Profit		
	$\eta_2 = 0.65$ $\eta_3 = 0.59$ $\delta_1 = 0.65$ $\delta_2 = 0.49$ $\eta_1 = 0.47$ $\alpha_2 = 0.69$ $\alpha_3 = 0.55$ $k_0 = 950$ $k_1 = 120$ $k_2 = 90$	$\eta_2 = 0.69$ $\eta_3 = 0.62$ $\delta_1 = 0.55$ $\delta_2 = 0.49$ $\eta_1 = 0.45$ $\alpha_2 = 0.72$ $\alpha_3 = 0.56$ $k_0 = 900$ $k_1 = 150$ $k_2 = 80$	$\eta_2 = 0.61$ $\eta_3 = 0.55$ $\delta_1 = 0.54$ $\delta_2 = 0.39$ $\eta_1 = 0.35$ $\alpha_2 = 0.75$ $\alpha_3 = 0.59$ $k_0 = 1000$ $k_1 = 100$ $k_2 = 50$
0.1	3.59827	4.17309	3.80843
0.2	3.67789	4.37047	3.85116
0.3	3.83764	4.68783	3.99023
0.4	4.08536	5.15737	4.23971
0.5	4.44261	5.84607	4.63524
0.6	4.95729	6.89995	5.2584
0.7	5.739	8.68624	6.31344
0.8	7.08058	12.4277	8.45213
0.9	10.0689	25.9994	15.3405
1.0	11.1245	30.3421	20.4563

From Table 1, it has been observed that as failure rate " η_1 " increases, keeping the other parameters $\eta_2, \eta_3, \delta_1,$

$\delta_2, \alpha_1, \alpha_2$ and α_3 fixed, mean time to system failure decreases. This implies that expected life of the system decreases as failure rate increases. Thus we conclude that mean time to system failure can be increased by reducing failure rate. From Table 2, it has been seen that as repair rate " α_1 " increases, keeping the values of other parameters $\eta_2, \eta_3, \delta_1, \delta_2, \alpha_1, \eta_1$ and α_3 fixed, the mean time to system failure increases. This implies that expected life of the system increases with increasing repair rate. Further, from Table 3, it has been observed that profit of the model decreases as failure rate " η_1 " increases, irrespective of the other parameters $\eta_2, \eta_3, \delta_1, \delta_2, \alpha_1, \alpha_2, \alpha_3, k_0, k_1$ and k_2 . Hence it can be concluded that expected life of the system can be increased by reducing failure rate. Also from Table 4, it has been noticed that the profit of the model increases as repair rate " α_1 " increases, keeping the other parameters $\eta_2, \eta_3, \delta_1, \delta_2, \alpha_1, \eta_1$ and α_3 fixed. This implies that expected life of the system increases as repair rate increases.

10- Conclusion

In this paper, the profit analysis of the model comprising of three non-identical units has been obtained. The reliability technique of priority of one unit over other has been used. Expressions for various important reliability measures have been obtained using regenerative point technique. The numerical illustration reveals that MTSF and profit of the system decreases with increase in the failure rate. Further MTSF and profit of the model increases as repair rate increases. Thus we conclude that expected lifetime of the model can be enriched by either increasing the repair rate of the unit or by reducing the failure rate of the unit which in turn will enhance the performance and hence efficiency of the model. The study advocates that some preventive measures should be taken to avoid failure of the system, for that the priority strategy of reliability measure should be taken into consideration.

The results obtained in this research paper are useful for system planners who are engaged in designing highly reliable system. This is significant because it considers an actual problem and provides a unique conceptual model as a starting point for organizations to foster higher levels of engagement and check the efficiency and effectiveness of the system. It is Novel Research and good topic for research for manufacturers, designers and users.

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