Optimization of Random Phase Updating Technique for Effective Reduction in PAPR, Using Discrete Cosine Transform

Babak Haji Bagher Naeeni*
Department of Electrical Engineering, IRIB University, Tehran, Iran
bnaeeni@yahoo.com

Received: 01/Dec/2015 Revised: 12/Apr/2016 Accepted: 25/May/2016

Abstract

One of problems of OFDM systems, is the big value of peak to average power ratio. To reduce it, any attempt have been done amongst which, random phase updating is an important technique. In contrast to paper, since power variance is computable before IFFT block, the complexity of this method would be less than other phase injection methods which could be an important factor. Another interesting capability of random phase updating technique is the possibility of applying the variance of threshold power. The operation of phase injection is repeated till the power variance reaches threshold power variance. However, this may be a considered as a disadvantage for random phase updating technique. The reason is that reaching the mentioned threshold may lead to possible system delay. In this paper, in order to solve the mentioned problem, DCT transform is applied on subcarrier outputs before phase injection. This leads to reduce the number of required carriers for reaching the threshold value which results in reducing system delay accordingly.

Keywords: Orthogonal Frequency Division Multiplexing (OFDM); Peak-to-Average Power Ratio (PAPR); Random Phase Updating; Discrete Cosine Transform.

1. Introduction

If OFDM subcarriers would be summed in an inphase manner, a high signal peak would be produced in the time domain. As the signal variety range is an important factor in performance of telecommunication facilities (e.g. Transmitter/ Receiver), reduction in this range would lessen the system implementation expenses (1). High signal peaks in time domain can be troublesome in two aspects. One problem is reduction in analog to digital converters performance by expanding their input range and increasing the quantization noise. The other problem is a drop in performance level of RF amplifier in the transmitter side. In case of higher values of peak to average ratio of power in transmitted signal, trying to reduce the average values before the transmission process will affect the performance of amplifier and also raises the error probability in receiver; otherwise transmitted signal would be distorted because the peak signal power would be higher than the linear range of amplifier. Recently, PAPR overheads have been really considered as a problem, since the OFDM has had an important role in communication systems. In the last decade many solutions, such as Coding methods [3], Tone-reservation[4], Selective Mapping [5], Random phase injection algorithm [6] etc., has been proposed to solve the PAPR problem in OFDM systems. In this article we tended to improve the random phase updating technique (which is one of the phase injection methods) using discrete cosine transform as novel technique.

2. PAPR Problem and Random Phase Updating Technique

An important issue in multi carrier systems, especially OFDM systems, is the high ratio of peak to average power (PAPR), which is one of the inherent characteristics of the transmitted signal.

Generally, high peaks can be seen in transmitted signal when the phase of subcarriers signals would be summed constructively. PAPR is defined mathematically in the following review.

An OFDM signal can be written as follows [6]:

$$s(t) = \sum_{i=-\infty}^{+\infty} \sum_{m=0}^{M-1} b_m(i) e^{j2\pi({}^{m}/_{T})(t-iT)} p(t-iT) \qquad (1)$$

Where T is the OFDM symbol duration $b_m(i)$is the symbol of the mth sub channel at time interval iT. Which is ±1 for BPSK modulation, $p(t)$ is a rectangular function with amplitude one and duration T, and M is the number of carriers. The OFDM signal of (1) in the time interval of $0 \le t \le T$can be written as

$$s(t) = \sum_{m=0}^{M-1} b_m\, e^{j2\pi({}^{m}/_{T})t} \qquad (2)$$

The power of $s(t)$is:

$$p(t) = |s(t)|^2 = \sum_{m=0}^{M-1} \sum_{n=0}^{M-1} b_m\, b_n{}^{*} e^{j(2\pi(m-n)/T)t} \qquad (3)$$

* Corresponding Author

The PAPR of the OFDM signal is written as:

$$\mathrm{PAPR} = \frac{\mathrm{Max}\{p(t)\}}{\mathrm{Mean}\{p(t)\}} \quad (4)$$

The variation of the instantaneous power of OFDM signal from the average is:

$$\Delta p(t) = p(t) - E[p(t)] \quad (5)$$

And accordingly, the power variance (PV) of OFDM signal, denoted by p, can be written as [3]:

$$\rho = \frac{1}{T}\int_0^T \left(\Delta p(t)\right)^2 dt = \sum_{i=1}^{M-1} |R_{bb}(i)|^2 \quad (6)$$

Where $R_{bb}(i)$ is the autocorrelation function of the sequenceb_m.

$$R_{bb}(i) = \sum_{m=0}^{M-1-i} b_m b_{m+i}{}^{*} \quad (7)$$

The power variance p is a good measure of the PAPR. PV and PAPR are related to each other according to the following relationship:

$$Q\left(\frac{\mathrm{PAPR}-1}{\sqrt{\rho}}\right) + Q\left(\frac{1}{\sqrt{\rho}}\right) = \beta \quad (8)$$

Where denotes the probability that $p(t)$ be less than or equal to pmax and

$$Q(y) = \frac{1}{\sqrt{2\pi}}\int_y^{\infty} e^{-u^2/2}\, du \quad (9)$$

From (7) it is seen that for a fixed β the OFDM signal with high PAPR has a high value of ρ**.**Because of the less computational burden in calculation ofρ, [see (5), 6)], in this method concentrate on the power (variance and assess its value for the random phase updating algorithm. However, using (7) the corresponding value of PAPR can also be obtained As shown in Fig. 1 in the random phase updating algorithm for each carrier a random phase is generated and assigned to that carrier. Using (2) the OFDM signal with phasing is written as:

$$s(t) = \sum_{m=0}^{M-1} b_m\, e^{j2\pi\left(\left({}^{m}/_{T}\right)t+\emptyset_m\right)} \quad (10)$$

Where $2\pi\emptyset_m$ is the mth subcarrier phase shift. Adding random phases to each subcarrier will change the power variance of OFDM signal. In the random phase updating algorithm, the phase of each subcarrier is updated by a random increment as:

$$(\emptyset_m)_i = (\emptyset_m)_{i-1} + (\Delta\emptyset_m)_i m = 0,1,\ldots,M-1 \quad (11)$$

Where iis the iteration index and $(\Delta\emptyset_m)_i$ is the phase increment of mth subcarrier at ith iteration. In the random phase updating method, without loss of generality, the initial phase, i.e., $(\Delta\emptyset_m)_0$, can be considered zero. Consequently, a random phase increment is generated and the phase is updated by adding the increment to the phase of that subcarrier. Flow chart of the algorithm for this iterative phase updating is shown in Fig. 2. In Fig. 2(A) a certain threshold for PV is set and for Fig.2 (B) a limited number of iterations is allowed:

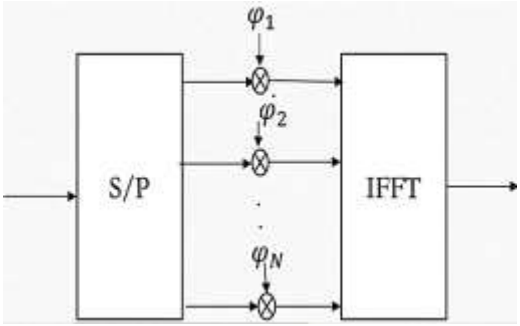

Fig. 1. Block diagram of OFDM with phasing showing the principle of adding phase shifts to the OFDM symbols.

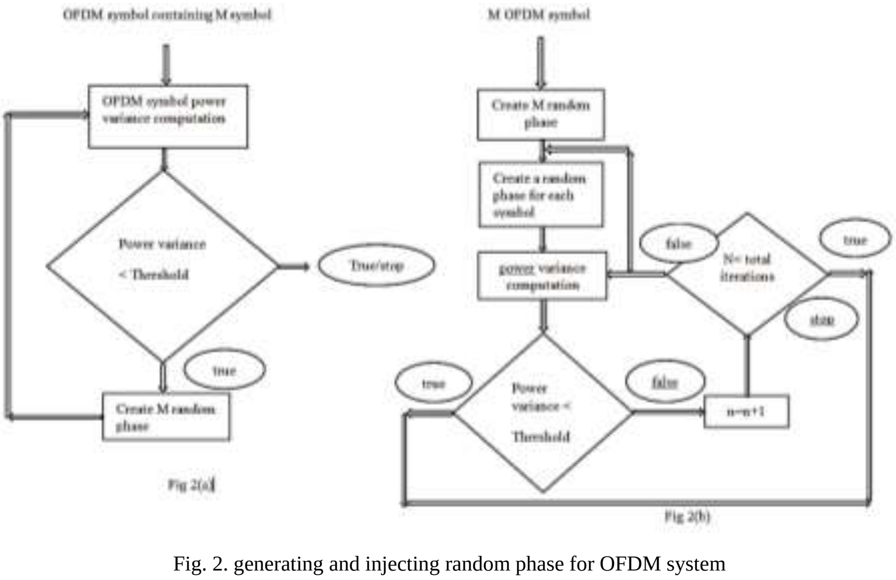

Fig. 2. generating and injecting random phase for OFDM system

In this method different distributions for the random phase increments have been considered and their influence on the PV has been investigated. Two distributions are Gaussian ($\Delta\emptyset_m = N(0, x^2)$) and Uniform ($\Delta\emptyset_m = \mathrm{Unif}[0, x]$), where $x \in \{0, 0.25, 0.5, 0.75\}$. Results are shown in Table I.

Table 1. power variance and number of iterations for the random phase updating algorithm with uniform and Gaussian distributions of phase increments

	Uniform Distribution				Normal Distribution			
	Power Variance		No.of iteration		Power variance		No.of iteration	
x	Mean	Std.dv	Mean	Std.dv	Mean	Std.dv	Mean	Std.dv
0.1	71.89	9.02	33.38	42.5	72.12	8.97	39.05	**50**
0.25	68.85	9.59	58.56	10.6	69.08	9.57	9.54	**11.8**
0.5	67.57	9.99	4.93	6.16	67.64	9.92	5.15	**6.39**
0.75	67.47	10	4.62	5.75	67.45	10	4.67	**5.81**
1.0	67.41	10	4.64	5.8	67.36	10	4.64	**5.76**

It is seen that there is no significant difference in the PV results when Gaussian or Uniform distribution is considered for the phase increments. In the rest of the method the uniform distribution has been chosen for the distribution of phase increments. The influence of different variances of the phase increments on the reduction of OFDM signal has been investigated. Results indicate a connection between phase shift variance and the PV number of iterations required reaching the threshold. Simulations have been carried out for different number of carriers as well as different PV thresholds. As shown in Fig. 3 when variance of phase shift increments is small more number of iterations is required.

This can be clearly justified. When standard deviation of phase increments is small the generated phases are likely not good to reduce the PAPR. But when the standard deviation of phase increments is large, the random phase increments have larger variations and it is more likely that their values be proper to decrease the PV. As seen in Fig. 4 by increasing the standard deviation of phase increments the number of iterations to reach the threshold decreases. Meanwhile, the lower the PV threshold the more the number of iterations. That is quite clear since lower threshold or smaller PV needs more iterations to select the proper phases for the subcarriers. From Fig. 3 the influence of different number of carriers on the number of iterations for different variances of phase shifts is also clear. It is obvious that increasing number of carriers from 8 to 48 slightly changes the number of iterations of the algorithm. As shown in Fig. 4, and unlike the number of carriers, the threshold level has a significant effect on the number of iterations of the algorithm. Efficiency of the algorithm is mainly related to the selected threshold level and consequently number of iterations and not the number of carriers. This is why in Section IV the dynamic reduction of threshold is proposed.

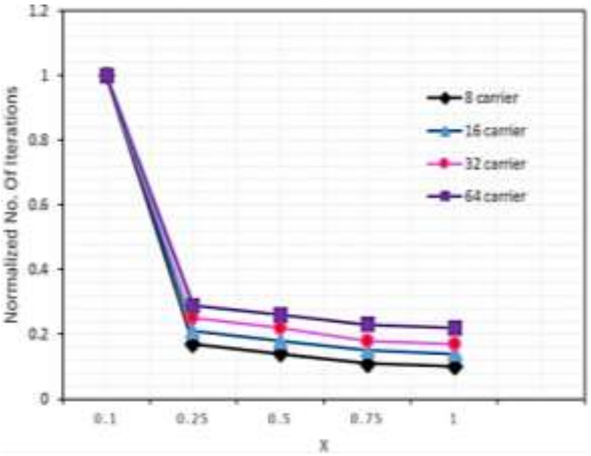

Fig. 3. Normalized mean number of iterations versus phase shift variance parameters a; for M = {8, 16, 32, 48} BPSK OFDM signal simulated with random phase updating algorithm (Fig. 2(A)).

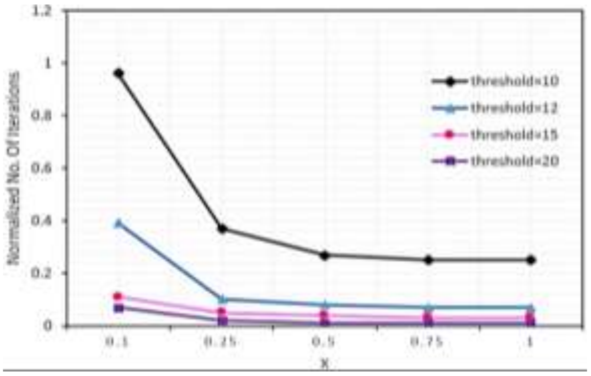

Fig. 4. Normalized mean number of iterations versus phase shift variance parameter a; for 8-carrier OFDM system and different threshold levels, simulated with random phase updating algorithm of Fig. 2(A).

Reducing of the PAPR with phasing implies a high degree of complexity and side information. For large number of carriers the computational burden for the calculation of PV is increased [see (5)]. Besides, because of more carriers more phases are involved in the algorithm which leads to more side information. The phase shifts have to be known at the transmitter and receiver. To lessen the problem, the quantization and grouping of the random phase increments has been carried out. Quantization of the phase shift to BPSK (i.e., 2) or QPSK (i.e., 4) type phase shifts (i.e., $\Delta\emptyset_m \in \{0, 0.5\}$ or $\{0, 0.25, 0.5, 0.75\}$, respectively) each phase shift which leads to a reduced complexity of the algorithm. Grouping means subcarriers are bundled and all subcarriers in the same bundle (group) get the same phase shift increment (see Fig. 5). By grouping the complexity of the algorithm is further reduced. Simulations were carried out for a 16-carrier OFDM for two and four levels of phase quantization and different number of iterations (Fig. 6). Results shown in Fig. 6 indicate that rounding of the phase increments to two levels does not change the variance and reduces the mean of PV. Grouping for 16 carrier BPSK-OFDM was examined with

2 groups of 8 carriers, 4 groups of 4 carriers and 8 groups of 2 carriers and for different

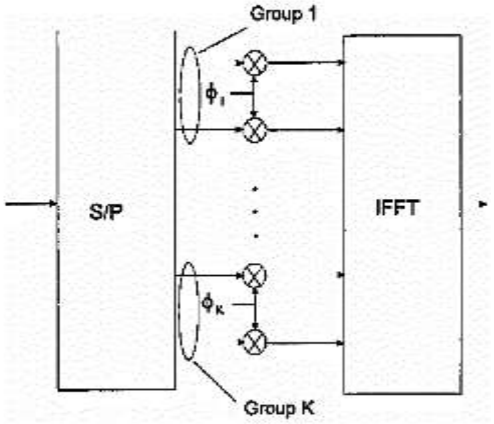

Fig. 5. Block diagram illustrating the grouping of the phases.

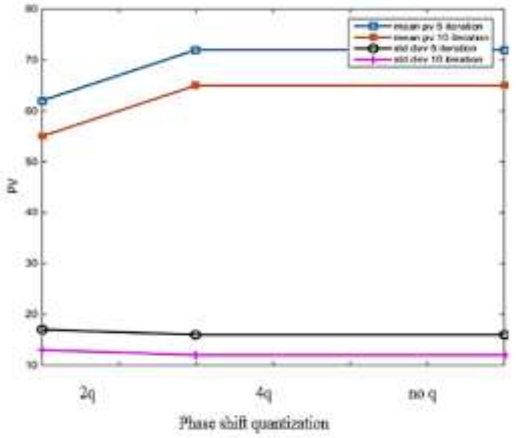

Fig. 6. Mean and standard deviation of power variance versus the phase quantization level for different number of iterations. OFDM with 16 carriers and BPSK modulation.

3. Discrete Cosine Transform

Discrete cosine transform is a reversible transform [7]. Input of this transform should be a vector and the process returns it's discrete cosine Fourier transform as an output. This way autocorrelation of the input series would be dropped so that the PAPR would be reduced [8].

Discrete cosine transform is defined as equation (12):

$$y(k) = w(k) \sum_{n=1}^{N} x(n) \cos\left(\frac{\pi(n-1)(k-1)}{2N}\right), \quad k = 1,2 \dots, N \tag{12}$$

Which in equation:

$$w(k) = \begin{cases} \frac{1}{\sqrt{N}}, & k = 1 \\ \sqrt{\frac{2}{N}}, & 2 \leq k \leq N \end{cases}$$

Since the discrete cosine transform reduces the inter-symbol correlation, it can also reduce the PAPR. This transform is really interested because of it produces low complexity and also it has no destructive effects on BER [9].

4. Combination of Discrete Cosine Transform and Updating Random Phase Techniques

One of the important advantages of random phase updating technique, is the relation between OFDM symbol power variance and PAPR. The complexity of this technique is less than that of most phase injection methods, because opposed to PAPR method, the power variance is computable before IFFT block. Another capability of random phase updating technique, is applying threshold power variance. In this method, phase injection would be continued until the time that power variance reaches to its threshold. However, it is possible that the power would not reach to its threshold value in a pre-determined time. This could be a disadvantage for random phase updating technique.

Therefore another technique is required as well as random phase updating for reducing both PAPR and complexity, currently. It is worth mentioning that in the proposed system, DCT of subcarriers is calculated before phase injection. This results in reducing symbols correlation which leads to power spectral density reduction. After phase injection, random phase updating is applied to new subcarriers. This leads to more reduction of PAPR compared to random phase updating. Also, the number of iterations for reaching threshold power variance world be would reduced. The block finally the BER would be reduced.

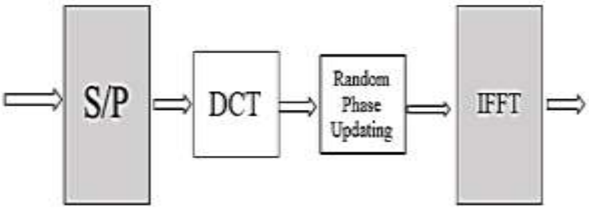

Fig. 7. The block diagram of combining random phase updating and DCT

5. Simulation Results

In the simulation results, it will be shown that the number of iterations required for reaching threshold power variance, PAPR value and bit error rate parameter, would be improved using the proposed combination technique.

5.1 Reducing the Number of Iterations Required for Reaching Threshold Power Variance

As discussed in section 4, reaching a desired threshold power variance requires to additional delay in random phase updating technique. This is one of the most important disadvantages of the mentioned technique. In order to, solve this problem, simultaneous using both DCT pre-coder and random phase updating is proposed in this paper. The proposed scheme is simulated using MATLAB. As it is shown in pictures (8 and 9), applying discrete cosine transform besides updating random phase technique, causes a considerable reduction in the numbers of repetitions required to reach the power variance (rather than updating random phase technique itself).

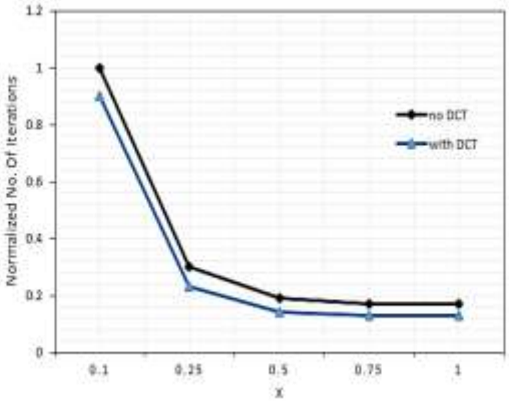

Fig. 8. Effect of applying DCT in numbers of repetitions required to reach the power variance in an 8 subcarrier OFDM system.

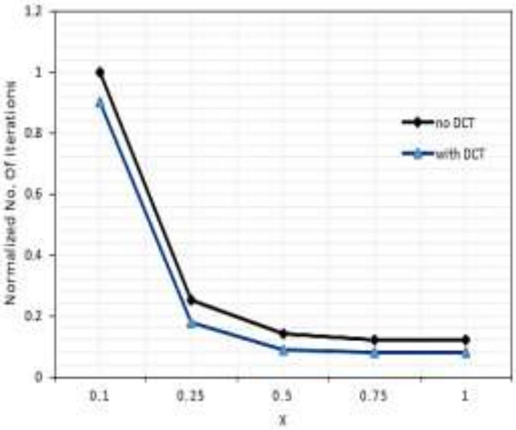

Fig. 9. Effect of applying DCT in numbers of repetitions required to reach the power variance in a 32-subcarrier OFDM system.

As shown in pictures (10-13), discrete cosine transform and updating random phase technique together, reduces the numbers of repetitions required to reach the power variance. Simulation has been done on a 16-carrier OFDM system using BPSK modulation for a 10 dB power variance (Picture (10)); a 12 dB power variance (Picture (11)); a 15 dB power variance (Picture (12)) and a 22 dB power variance (Picture (13)).

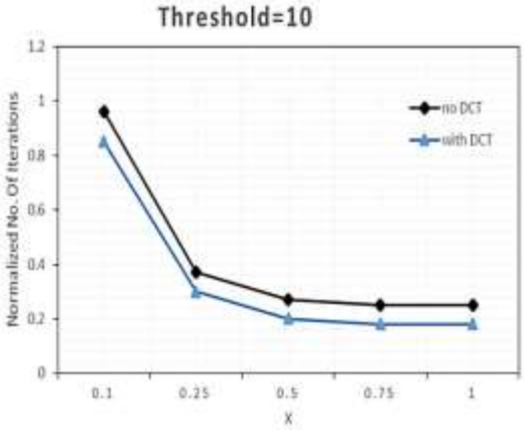

Fig. 10. Effect of applying DCT besides updating random phase technique in numbers of repetitions required to reach the power variance of 10 dB in a 16-subcarrier OFDM system using BPSK modulation.

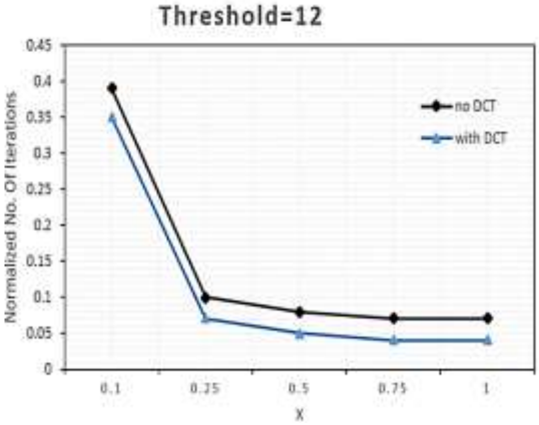

Fig. 11. Effect of applying DCT besides updating random phase technique in numbers of repetitions required to reach the power variance of 12 dB in a 16-subcarrier OFDM system using BPSK modulation.

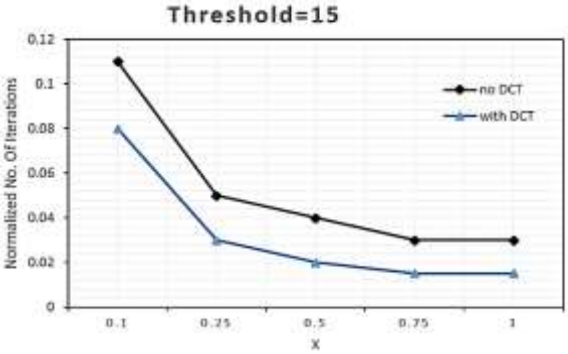

Fig. 12. Effect of applying DCT besides updating random phase technique in numbers of repetitions required to reach the power variance of 12 dB in a 16-subcarrier OFDM system using BPSK modulation.

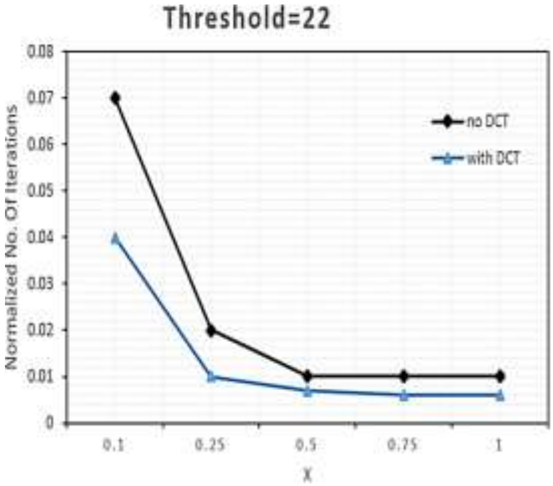

Fig. 13. Effect of applying DCT besides updating random phase technique in numbers of repetitions required to reach the power variance of 22 dB in a 16-subcarrier OFDM system using BPSK modulation.

5.2 Reducing PAPR Value

Based on (7), power variance value has a forward correlation with PAPR. Therefore, using the proposed combination scheme, PAPR would be reduced with a value of 1.8 db more than that of random phase updating method. This, can be verified using CCDF diagram.

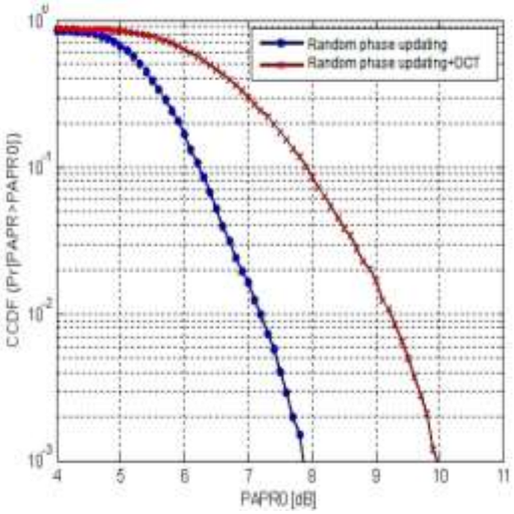

Fig. 14. Comparison between reducing PAPAR in two methods.

5.3 Improving BER

In order to send a signal through a wireless communication channel, it is required that the transmitting signal be amplified at the sender side. It should be mentioned that the power amplifiers often have a limited range. Therefore, the signal power could not be increased with a desired value. As a result, BER at the receiver side would be increased due to signal attenuation. However, since using the proposed combination scheme PAPR would be decreased, BER would be decreased too. Based on Fig.15, BER of the proposed scheme decreased with a value of 2.1 db compared to random phase updating technique.

6. Conclusion

In the proposed scheme, the DCT of subcarriers should be calculated before phase injection. Then, random phase updating should be applied to new sub-carriers. The result of this modification is as follows a. reduction of the numbers of iterations required to reach the threshold power variance, b. reducing PAPR with a value of 1.8 db, compared to random phase updating technique. And c. improvement of BER.

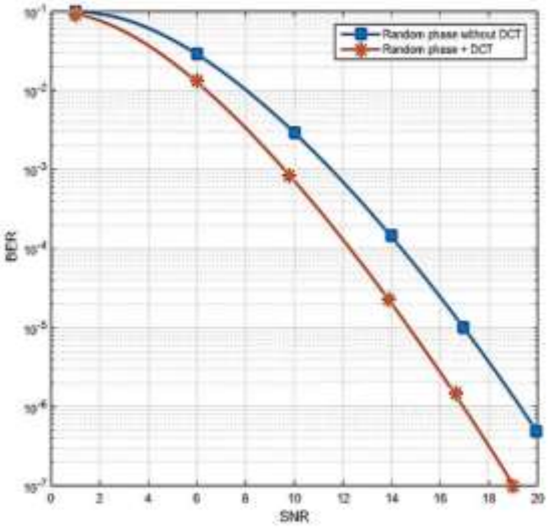

Fig. 15. BER reduction in an OFDM system using the proposed combination scheme.

References

[1] V. K. Singh, A. Goel, A. Sharma. " Reducing Peak to Average Power Ratio of OFDM by Using Selective Mapping". International Journal of Research in Information Technology, vol. 2, No.4, pp. 400-407, April 2014.

[2] T. Jiang and G. Zhu, "Complement block coding for reduction in peak to average power ratio of OFDM signals." IEEE Communications Magazine, vol. 43, no. 9, pp. s17-s22, Sept. 2005.

[3] Md. Ibhrahim Abdullah et.al, "Comparative Study of PAPR Reduction Techniques in OFDM". ARPN journal of system and software, vol. 1, no. 8, pp. 263-269, nov 2011.

[4] P. Phoomchusak, C. Pirak, "Adaptive tone-reservation PAPR technique with optimal subcarriers allocation awareness for multi-user OFDMA systems," in Proc. ICACT, 2011, pp.814-818.

[5] H. Chen, H. Liang. "Combined Selective Mapping and Binary Cyclic Codes for PAPR Reduction in OFDM Systems." IEEE Transactions on Wireless Communications, vol.6, pp.3524-3528, Oct. 2007.

[6] Nikookar, H., and Lidsheim, k.S.,"Random phase updating algorithm for OFDM transmission with low PAPR." IEEE Transaction on Broadcasting, Vol.48, Jun 2002.

[7] H. Ochiai and H. Imai, "Performance of the deliberate clipping with symbol selection for strictly band-limited OFDM systems." IEEE J. Sel. Areas Commun., vol. 18, pp.123-128, Nov. 2000.

[8] Jayalath, A., Tellambura, C., "Adaptive PTS approach for reduction of peak-to-average power ratio of OFDM signal." Electronics Letters, Vol.36, pp. 1226-1228, 2000.

[9] S. Gazor, R. AliHemmati, "Tone reservation for OFDM systems by maximizing signalto-distortion ratio." IEEE Transactions on Wireless Communications,Vol.11,pp. 762-770, 2012.

Babak Haji Bagher Naeeni received the B.Sc and M.Sc in electrical engineering from central Tehran branch, Islamic Azad university, Tehran, Iran, and from south Tehran branch, Islamic Azad university, Tehran, Iran, in 1993 and 1999, respectively. He is assistant professor of electrical engineering at IRIB university. His research interests include multiuser detection, channel equalization, artificial intelligence.