Joint Relay Selection and Power Allocation in MIMO Cooperative Cognitive Radio Networks

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Abstract

In this work, the issue of joint relay selection and power allocation in Underlay MIMO Cooperative Cognitive Radio Networks (U-MIMO-CCRN) is addressed. The system consists of a number of secondary users (SUs) in the secondary network and a primary user (PU) in the primary network. We consider the communications in the link between two selected SUs, referred to as the desired link which is enhanced using the cooperation of one of the existing SUs. The core aim of this work is to maximize the achievable data rate in the desired link, using the cooperation of one of the SUs which is chosen opportunistically out of existing SUs. Meanwhile, the interference due to the secondary transmission on the PU should not exceed the tolerable amount. The approach to determine the optimal power allocation, i.e. the optimal transmits covariance and amplification matrices of the SUs, and also the optimal cooperating SU is proposed. Since the proposed optimal approach is a highly complex method, a low complexity approach is further proposed and its performance is evaluated using simulations. The simulation results reveal that the performance loss due to the low complexity approach is only about 14%, while the complexity of the algorithm is greatly reduced.

Keywords: Cognitive Radio Networks; Cooperative Communications; MIMO Systems; Low Complexity Approach.

1. Introduction

Since the issuance of the report of Federal Communications Commission (FCC) in 2002, which revealed the spectrum inefficiency in the incumbent wireless communication systems, cognitive radio (CR) has been regarded as one potential technology to activate the utilization of spectrum resources in the recent evolution of wireless communication systems [1]. As a consequence, the overlay and underlay modes can be developed, based on the definitions of spectrum holes in [1] and the operation modes in [2, 3], to use the white and gray spectrum holes, respectively.

To further enhance the system performance, a cooperative relay network can be incorporated into secondary system (SS). Thus, in the underlay CR system with an IT limit, the cooperative relay networks can also be applied to have a better capacity and error rate performance [5], trade-off between achievable rate and network lifetime [6], maximum signal-to-interference-plus-noise ratio (SINR) at the destination node [7], better channel utilization by multi-hop relay [8], maximum throughput and reduced interference via beam forming [9], and maximum SINR using cooperative beam forming [10].

Multiple-input/multiple-output (MIMO) systems have a great potential to enhance the throughput in the framework of wireless networks [11, 12]. Using M transmits antennas at the transmitter and N receive antennas at the receiver, the capacity of a MIMO single user is equal to $m\ \{M, N\}$ times the capacity of a single-input/single-output (SISO) system [11, 12]. Multiple antennas can be applied to achieve many desirable goals, such as capacity increase without bandwidth expansion, transmission reliability enhancement via space-time coding, and co-channel interference suppression for multi-user transmission.

The method on relay selection and channel allocation in [13] greedily searches the most profitable pair to maximize system throughput, without considering the interference with primary users, which is the case for CR networks. The problem of joint relay selection and power allocation to maximize system throughput with limited interference to licensed (primary) users in cognitive radio networks was investigated in [14]. In [15], the structure of an optimal relay precoder design for Amplify-and-Forward based Underlay MIMO cognitive relay was studied.

Joint problems of relay selection and resource allocation in CR networks (CRNs) have attracted extensive

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research interests due to its more effective spectrum utilization [13]-[18]. The authors in [13] consider a cooperative cognitive radio network (CCRN) in which the relays are selected among the existing SUs. For CCRNs with decode-and-forward strategy, two relay selection schemes, namely, full-channel state information (CSI)-based best relay selection (BRS) and a partial CSI-based best relay selection (PBRS) were proposed in [14]. In order to obtain an optimal subcarrier pairing, relay assignment and power allocation in MIMO-OFDM based CCRNs, the dual decomposition technique was recruited in [15] to maximize the sum rate subject to the interference temperature limit of the PUs. The issue of joint relay selection and power allocation in two-way CCRN was considered in [16]. A suboptimal approach for reducing the complexity of joint relay selection and power allocation in CCRN was proposed in [17]. The network coding opportunities was exploited in [18].

The issue of resource allocation in MIMO CRNs was explored in [19]-[22]. The authors in [19] presented a low complexity algorithm for resource allocation in MIMO-OFDM based CR networks, using game theory approach and the primal decomposition method. In [20], the authors extended the pricing concept to MIMO-OFDM based CR networks and presented two iterative algorithms for resource allocation in such systems. To obtain an optimal subcarrier pairing, relay assignment and power allocation in MIMO-OFDM based CCRNs; the dual decomposition technique was recruited in [21] to maximize the sum-rate subject to the interference temperature limit of the PUs. Moreover, due to high computational complexity of the optimal approach, a suboptimal algorithm was further proposed in [21] and [22].

In this paper, we consider the opportunistic spectrum access in MIMO cognitive radio networks (MIMO-CRN). More specifically, we propose a Cognitive Cooperative communication protocol based on Beam forming (CCB) in MIMO-CRN which ensures the SU's continuous transmission and reduces its outage probability without interfering the PUs. The desired link is considered as the MIMO link between two SUs, the SU TX and SU RX. Meanwhile, CCB adopts beam forming at the SU RX and the cooperating SU. As a result, the SU RX only receives signals from the SU TX and the best relay, and the interferences from the PUs are suppressed. The same story applies to the cooperating SU as a result of beam forming. To be more accurate, when a PU transmits signal in the system, the joint problems of opportunistic relay selection and power allocation in the context of MIMO CR networks to maximize the end-to-end achievable data rate of Underlay MIMO CR networks need to be considered. Our focus is on the amplify-and-forward (AF) relay strategy. An obvious reason is that AF has low complexity since no decoding/encoding is needed. This benefit is even more attractive in MIMO-CRN, where decoding multiple data streams could be computationally intensive. In addition to simplicity, a more important reason is that AF outperforms decode-and-forward (DF) in terms of network capacity scaling: in general, as the number of relays increases in MIMO-CRN, the effective signal-to-noise ratio (SNR) under AF scales linearly, as opposed to being a constant under DF [30].

The remainder of this paper is organized as follows. Section 2 presents the system model and general formulation of the problem. In Sections 3, the structure of optimal power allocation matrices is studied. Based on these structural results, we simplify and reformulate the optimization problem. The optimization algorithms, including the optimal and suboptimal approach are discussed in Section 4. In Section 5, the outage probability of the desired link is analyzed. Numerical results are provided in Section 6 to show the efficacy of the proposed algorithms and Section 7 concludes this paper.

Notation: The following notation is used throughout the paper. The operators $(.)^H$, $|.|$, $Tr(.)$ and $(.)^+$ are Hermitian (complex conjugate), determinant, trace and pseudo-inverse operators, respectively.

2. System Model

We consider a scenario where a CR network, consisting of $N_{SU}+2$ SUs, coexists with a primary network, consisting of N_{PU} PU pair. In this paper the communication between two SUs is considered, which is also referred to as the desired SU link. The SU transmitter (SU TX) transmits signals to SU receiver (SU RX) either in the direct link or taking advantage of the cooperation of one of the SUs, depending on the presence of the PUs in the system. When the PUs are absent, the SU TX simply communicates the SU RX directly. Therefore, throughout this and next sections, we assume that the PU pairs are present and, as discussed in the previous section, it is inevitable for the SU TX to take advantage of the cooperation of one the SUs to keep the imposed interference on the PUs in the allowed region.

2-1- The transmission process at the presence of PUs

When the PU pairs are present, the direct communications between the SU TX and SU RX may impose intolerable interference on the PUs. The cooperation of one of SUs with the desired SU link can provide the possibility of reducing the transmit power of the SUs and thereby less interference is imposed on the PU pairs. A transmission from SU TX to SU RX at the presence of PUs takes two time-slots. In the first time-slot, the SU TX transmits signals to all the existing SUs in the CR network and the SUs employ beamforming to only receive signal of the SU TX. In the second time-slot, one of the SUs is selected to cooperate with the SU TX by amplifying its received signal and forwarding it to the SU RX, without decoding the message. All the transmissions in the SU system need to be regulated in order to avoid excessive interference on the PU pair. Meanwhile, the interference from the PUs in the SU TX is avoided by employing beam forming. The set of candidate SUs to cooperate with the desired SU link is denoted by S_R . Besides, the set of PU pairs is also denoted by S_{PU} . It is further assumed that all the users, including the SUs and the PUs are equipped with multiple-antennas. Without loss of generality and for ease of exposition, we assume that the entire candidate SUs to cooperate with desired link are equipped with N_r antennas and the PUs with N_p antennas. The number of antennas at SU TX and SU RX are also N_s and N_d, respectively. $\mathbf{H}_{sr,i} \in \mathbb{C}^{N_r \times N_s}$ represents the channel matrix form SU TX to SU *i* and $\mathbf{H}_{rd,i} \in \mathbb{C}^{N_r \times N_d}$ represents the channel form SU *i* to SU RX. All the channels are modeled as Rayleigh fading channels and invariant during one time slot. It is further assumed that all the instantaneous channel matrices are perfectly known at the SU TX. The assumption of perfect knowledge of all the channel gains is a typical assumption in this area [31, 32]. In the presence of PUs, the amplify-and-forward (AF) relaying protocol is used.

2-2- Problem Formulation

The received signal at *i*-th SU can be written as

$$\mathbf{y}_{r,i} = \mathbf{H}_{sr,i}\mathbf{x}_{s,i} + \mathbf{n}_{r,i}, \quad \forall i \in S_R \qquad (1)$$

where the transmit signal of SU TX, intended for SU *i*, is denoted by $\mathbf{x}_{s,i} \in \mathbb{C}^{N_s \times 1}$. $\mathbf{n}_{r,i} \in \mathbb{C}^{N_r \times 1}$ is the additive white Gaussian noise at SU *i*. Note that in (1) the negative effect of the PU signal on received signal of the candidate SUs is canceled, due to employment of beam forming. Suppose that SU *i* is selected to cooperate with the desired SU link. Then, the received signal at SU RX from SU *i* is given by

$$\begin{aligned}\mathbf{y}_d &= \mathbf{H}_{rd,i}\mathbf{A}_i\mathbf{y}_{r,i} + \mathbf{n}_d \\ &= \mathbf{H}_{rd,i}\mathbf{A}_i\mathbf{H}_{sr,i}\mathbf{x}_{s,i} + \mathbf{H}_{rd,i}\mathbf{A}_i\mathbf{n}_{r,i} + \mathbf{n}_d\end{aligned} \qquad (2)$$

where $\mathbf{A}_i$ represents the amplification matrix, used at SU *i* ; $\mathbf{n}_d \in \mathbb{C}^{N_d \times 1}$ is the additive white Gaussian noise at SU RX. Once again, it is presumed that the interference from the PUs is eliminated at the SU RX, by recruiting the appropriate beam forming. As a result of cooperation of one of the SUs, SU *i*, the achievable data rate in the desired link can be written as

$$\begin{aligned}R_i = \frac{1}{2}\log_2\Big|&\mathbf{I}_{N_d} + \mathbf{H}_{rd,i}\mathbf{A}_i\mathbf{H}_{sr,i}\mathbf{Q}_i\mathbf{H}_{sr,i}^H\mathbf{A}_i^H\mathbf{H}_{rd,i}^H \\ &\times\left(\pm_d^2\mathbf{I}_{N_d} + \pm_r^2\mathbf{H}_{rd,i}\mathbf{A}_i\mathbf{A}_i^H\mathbf{H}_{rd,i}^H\right)^{-1}\Big|\end{aligned} \qquad (3)$$

where $\pm_r^2$ and $\pm_d^2$ denote the variances of $\mathbf{n}_{r,i}$ and $\mathbf{n}_d$, and $\mathbf{Q}_i$ denotes the transmit covariance matrix of SU TX, intended for SU *i*. The transmit power of SU TX is restricted to P_T, i.e. $Tr(\mathbf{Q}_i) \le P_T$.

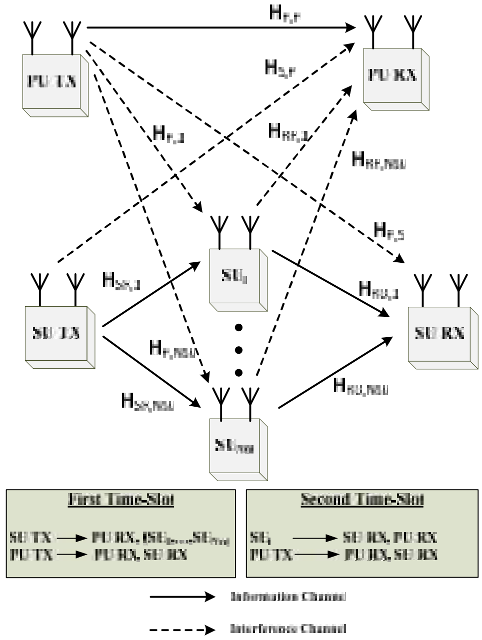

Fig. 1 System Model

Furthermore, the maximum transmit power of the SU *i*, if selected as the cooperative relay, is P_R. The PUs must not be disturbed as a result of transmission by SU TX and further the cooperation of the selected SU with the SU TX. In this way, the interference power constraints on the PUs are provided by $Tr\left(\mathbf{H}_{s,p,n}\mathbf{Q}_i\mathbf{H}_{s,p,n}^H\right) \le P_{I,1}$ and $Tr\left\{\mathbf{H}_{i,p,n}\left[\mathbf{A}_i\left(\pm_r^2\mathbf{I}_{N_r} + \mathbf{H}_{sr,i}\mathbf{Q}_i\mathbf{H}_{sr,i}^H\right)\mathbf{A}_i^H\right]\mathbf{H}_{i,p,n}^H\right\} \le P_{I,2}$, for all $n \in S_{PU}$, where the SU *i* is selected to cooperate with the SU TX. Moreover, $\mathbf{H}_{i,p,n}$ and $\mathbf{H}_{s,p,n}$ represent the channel from SU *i* and SU TX to *n*-th PU RX, respectively. Evidently, the maximum tolerable interference at the PUs is $P_{I,1} + P_{I,2}$. One of the aims of this work is to optimally select the cooperating SU and also calculate the optimum power allocation in the proposed system, which can be formulated as:

$$\begin{aligned}&\left(\mathbf{Q}_i^*, \mathbf{A}_i^*\right) = \arg\max_{\mathbf{Q},\mathbf{A}_i} \; R_i\left(\mathbf{Q}_i, \mathbf{A}_i\right) \\ &\qquad i = \arg\max_i \; R_i\left(\mathbf{Q}_i^*, \mathbf{A}_i^*\right) \\ &\text{s.t. } Tr\left(\mathbf{Q}_i\right) \le P_T \\ &\quad Tr\left[\mathbf{A}_i\left(\pm_r^2\mathbf{I}_{N_r} + \mathbf{H}_{sr,i}\mathbf{Q}_i\mathbf{H}_{sr,i}^H\right)\mathbf{A}_i^H\right] \le P_R \\ &\quad Tr\left(\mathbf{H}_{s,p,n}\mathbf{Q}_i\mathbf{H}_{s,p,n}^H\right) \le P_{I,1}, \quad \forall n \in S_{PU} \\ &\quad Tr\left\{\mathbf{H}_{i,p,n}\left[\mathbf{A}_i\left(\pm_r^2\mathbf{I}_{N_r} + \mathbf{H}_{sr,i}\mathbf{Q}_i\mathbf{H}_{sr,i}^H\right)\mathbf{A}_i^H\right]\mathbf{H}_{i,p,n}^H\right\} \le P_{I,2}, \forall n \in S_{PU} \\ &\quad \mathbf{A}_i \ge 0, \mathbf{Q}_i \ge 0\end{aligned} \qquad (4)$$

where $\mathbf{Q}_i^*$ and $\mathbf{A}_i^*$ are the optimum transmit covariance and amplification matrices. For convenience, we define two constraint sets according to the following:

$$\Phi_i \sqcup \left\{\mathbf{Q}_i \,|Tr\left(\mathbf{Q}_i\right) \le P_T\,, \; Tr\left(\mathbf{H}_{s,p,n}\mathbf{Q}_i\mathbf{H}^H_{s,p,n}\right) \le P_I\,, \mathbf{Q}_i \ge 0, \forall n \in S_{PU}\right\} \quad (5)$$

$$\Psi_i \sqcup \left\{\mathbf{A}_i \left| \begin{array}{l} Tr\left[\mathbf{A}_i\left(\pm_r^2\mathbf{I}_{N_r} + \mathbf{H}_{sr,i}\mathbf{Q}_i\mathbf{H}^H_{sr,i}\right)\mathbf{A}_i^H\right] \le P_R\,, \mathbf{A}_i \ge 0 \\ Tr\left\{\mathbf{H}_{i,p,n}\left[\mathbf{A}_i\left(\pm_r^2\mathbf{I}_{N_r} + \mathbf{H}_{sr,i}\mathbf{Q}_i\mathbf{H}^H_{sr,i}\right)\mathbf{A}_i^H\right]\mathbf{H}^H_{i,p,n}\right\} \le P_i \end{array}\right.\right\} \quad (6)$$

It is easy to verify that (4) can be *decomposed* into three parts as follows:

$$\max_{i \in S_R}\left(\max_{\mathbf{Q}_i \in \Phi_i}\left(\max_{\mathbf{A}_i \in \Psi_i} \; R_i\left(\mathbf{Q}_i, \mathbf{A}_i\right)\right)\right) \quad (7)$$

Hence, solving (4) reduces to *iteratively* solving a sub-problem with respect to A_i **,** for all $i \in S_R$ and $n \in S_{PU}$ (with $\mathbf{Q}_i$ fixed), then another sub-problem with respect to $\mathbf{Q}_i$ (with A_i fixed, $\forall i \in S_R$ and $n \in S_{PU}$) and finally a main problem with respect to i . Directly tackling problem (4) is intractable in general. However, we will exploit the inherent special structure to significantly reduce the problem complexity and convert it to an equivalent problem with scalar parameters. In what follows, we will first study the optimal structural properties of A_i and $\mathbf{Q}_i$. Based on these properties, we will reformulate (4).

3. Optimal Power Allocation in the SU TX and Cooperating SU

In the first subsection, the structure of the optimal amplification matrix in *i*-th SU for a given $\mathbf{Q}_i$ is investigated. Then, the optimal structure of $\mathbf{Q}_i$ is studied in second subsection. Finally, based on these optimal structures, the problem in (4) is reformulated in third subsection.

3-1- The Structure of the optimal amplification matrices

For now, we assume that $\mathbf{Q}_i$ is given. Let the eigenvalue-decomposition of $\mathbf{H}_{sr,i}\mathbf{H}^H_{sr,i}$ and $\mathbf{H}^H_{rd,i}\mathbf{H}_{rd,i}$ be

$$\mathbf{H}_{sr,i}\mathbf{H}^H_{sr,i} = \mathbf{U}_{sr,i}\Sigma_{sr,i}\mathbf{U}^H_{sr,i}\,, \; \mathbf{H}^H_{rd,i}\mathbf{H}_{rd,i} = \mathbf{V}_{rd,i}\Sigma_{rd,i}\mathbf{V}^H_{rd,i} \quad (8)$$

where $\mathbf{U}_{sr,i}$ and $\mathbf{V}_{rd,i}$ are unitary matrices, $\Sigma_{sr,i} = diag\left\{\sim_1, \sim_2, \ldots, \sim_{N_r}\right\}$ with $\sim_l \ge 0$, and $\Sigma_{rd,i} = diag\left\{\ _1, \ _2, \ldots, \ _{N_r}\right\}$ with $\ _l \ge 0$.

Proposition 1: *The optimal amplification matrix of SU i,* $\mathbf{A}_i$ *, has the following structure*

$$\mathbf{A}_{i,opt} = \mathbf{V}_{rd,i}\Lambda_{\mathbf{A}_i}\tilde{\mathbf{U}}^H_{sr,i} \quad (9)$$

Where $\tilde{\mathbf{U}}_{sr,i}$ *is obtained by eigenvalue decomposition of* $\tilde{\mathbf{H}}_{sr,i}\tilde{\mathbf{H}}^H_{sr,i}$ *and* $\tilde{\mathbf{H}}_{sr,i} = \mathbf{H}_{sr,i}\mathbf{Q}_i^{\frac{-1}{2}}$, *i.e.* $\tilde{\mathbf{H}}_{sr,i}\tilde{\mathbf{H}}^H_{sr,i} = \mathbf{H}_{sr,i}\tilde{\mathbf{Q}}_i\mathbf{H}^H_{sr,i} = \tilde{\mathbf{U}}_{sr,i}\tilde{\Sigma}_{sr,i}\tilde{\mathbf{U}}^H_{sr,i}$.

Proof. Please refer to appendix A.

Let the singular value decomposition (SVD) of $\mathbf{H}_{sr,i}$ and $\mathbf{H}_{rd,i}$ be

$$\mathbf{H}_{sr,i} = \mathbf{U}_{sr,i}\Lambda_{sr,i}\mathbf{V}^H_{sr,i}\,, \; \mathbf{H}_{rd,i} = \mathbf{U}_{rd,i}\Lambda_{rd,i}\mathbf{V}^H_{rd,i} \quad (10)$$

which satisfies (8). Then exploiting (9), (10) and (3), the achievable data rates of the desired link can be written as

$$\begin{aligned} R_i\left(\mathbf{Q}_i, \mathbf{A}_{i,opt}\right) = \\ \frac{1}{2}\log_2\Big|\mathbf{I}_{N_d} + \mathbf{H}_{rd,i}\mathbf{A}_{i,opt}\mathbf{H}_{sr,i}\mathbf{Q}_i\mathbf{H}^H_{sr,i}\mathbf{A}^H_{i,opt}\mathbf{H}^H_{rd,i} \\ \times\left(\pm_d^2\mathbf{I}_{N_d} + \pm_r^2\mathbf{H}_{rd,i}\mathbf{A}_{i,opt}\mathbf{A}^H_{i,opt}\mathbf{H}^H_{rd,i}\right)^{-1}\Big| \\ = \frac{1}{2}\log_2\left|\mathbf{I}_{N_d} + \Lambda^2_{rd,i}\Lambda^2_{\mathbf{A}_i}\tilde{\Sigma}_{sr,i}\left(\pm_d^2\mathbf{I}_{N_d} + \pm_r^2\Lambda^2_{rd,i}\Lambda^2_{\mathbf{A}_i}\right)^{-1}\right| \end{aligned} \quad (11)$$

According to (11), the achievable data rate in the desired SU link only depends on $\tilde{\Sigma}_{sr,i}$ but not on $\tilde{\mathbf{U}}_{sr,i}$. Then, it can be concluded that for any matrix $\hat{\mathbf{Q}}_i$ which satisfies $\mathbf{H}_{sr,i}\hat{\mathbf{Q}}_i\mathbf{H}^H_{sr,i} = \hat{\mathbf{U}}_{sr,i}\tilde{\Sigma}_{sr,i}\hat{\mathbf{U}}^H_{sr,i}$, the optimal data rate is the same as when the transmit covariance matrix in the desired link is $\tilde{\mathbf{Q}}_i$. Therefore (9) can be written as

$$\mathbf{A}_{i,opt} = \mathbf{V}_{rd,i}\Lambda_{\mathbf{A}_i}\mathbf{U}^H_{sr,i} \quad (12)$$

3-2- The Structure of the optimal transmit covariance Matrix

In this subsection, the optimal structure of the transmit covariance matrix of the desired link is determined.

Proposition 2: *The structure of optimal transmits covariance matrix of SU TX is as follow:*

$$\mathbf{Q}_i = \mathbf{V}_{sr,i}\Lambda_{\mathbf{Q}_i}\mathbf{V}^H_{sr,i} \quad (13)$$

where $\Lambda_{\mathbf{Q}_i}$ *is a diagonal matrix and must be determined such that the achievable data rate in the desired link is maximized.*

Proof. Suppose that $\tilde{\Sigma}_{sr,i,1}$ is $r \times r$, then

$$\begin{aligned} \mathbf{H}_{sr,i}\hat{\mathbf{Q}}_i\mathbf{H}^H_{sr,i} = \hat{\mathbf{U}}_{sr,i}\tilde{\Sigma}_{sr,i}\hat{\mathbf{U}}^H_{sr,i} \\ = \left[\hat{\mathbf{U}}_{sr,i,1} \quad \hat{\mathbf{U}}_{sr,i,2}\right]\begin{bmatrix}\tilde{\Sigma}_{sr,i} & \\ & \mathbf{0}\end{bmatrix}\left[\hat{\mathbf{U}}_{sr,i,1} \quad \hat{\mathbf{U}}_{sr,i,2}\right]^H \end{aligned} \quad (14)$$

where $\hat{\mathbf{Q}}_i$ is any PSD [1] matrix which satisfies $\mathbf{H}_{sr,i}\hat{\mathbf{Q}}_i\mathbf{H}^H_{sr,i} = \hat{\mathbf{U}}_{sr,i}\tilde{\Sigma}_{sr,i}\hat{\mathbf{U}}^H_{sr,i}$. Hence the singular value decomposition of matrix $\mathbf{H}_{sr,i}$ with rank r can be expressed as

1. Positive Semi-Definite

$$\mathbf{H}_{sr,i} = \mathbf{U}_{sr,i}\Lambda_{sr,i}\mathbf{V}_{sr,i}^{H} = \begin{bmatrix}\mathbf{U}_{sr,i,1} & \mathbf{U}_{sr,i,2}\end{bmatrix}\begin{bmatrix}\Lambda_{sr,i,1} & \\ & \mathbf{0}\end{bmatrix}\begin{bmatrix}\mathbf{V}_{sr,i,1} & \mathbf{V}_{sr,i,2}\end{bmatrix}^{H} \quad (15)$$

where $\Lambda_{sr,i,1}$ is $r \times r$. It can be shown that $\mathbf{U}_{sr,i,1}$ is orthogonal to $\hat{\mathbf{U}}_{sr,i,2}$. Moreover, $\mathbf{U}_{sr,i,2}$ is orthogonal to $\hat{\mathbf{U}}_{sr,i,1}$. The pseudo-inverse of $\mathbf{H}_{sr,i}$ is denoted by $\mathbf{H}_{sr,i}^{+}$. Then from

$$\mathbf{H}_{sr,i}\hat{\mathbf{Q}}_i\mathbf{H}_{sr,i}^{H} = \hat{\mathbf{U}}_{sr,i}\tilde{\Sigma}_{sr,i}\hat{\mathbf{U}}_{sr,i}^{H}$$

we have

$$\begin{aligned}&\mathbf{H}_{sr,i}^{+}\mathbf{H}_{sr,i}\hat{\mathbf{Q}}_i\mathbf{H}_{sr,i}^{H}\mathbf{H}_{sr,i}^{+H} = \\ &\begin{bmatrix}\mathbf{V}_{sr,i,1} & \mathbf{V}_{sr,i,2}\end{bmatrix}\begin{bmatrix}\Lambda_{sr,i,1}^{-1} & \\ & \mathbf{0}\end{bmatrix}\begin{bmatrix}\mathbf{U}_{sr,i,1} & \mathbf{U}_{sr,i,2}\end{bmatrix}^{H} \\ &\times\begin{bmatrix}\hat{\mathbf{U}}_{sr,i,1} & \hat{\mathbf{U}}_{sr,i,2}\end{bmatrix}\begin{bmatrix}\tilde{\Sigma}_{sr,i,1} & \\ & \mathbf{0}\end{bmatrix}\begin{bmatrix}\hat{\mathbf{U}}_{sr,i,1} & \hat{\mathbf{U}}_{sr,i,2}\end{bmatrix}^{H} \\ &\times\begin{bmatrix}\mathbf{U}_{sr,i,1} & \mathbf{U}_{sr,i,2}\end{bmatrix}\begin{bmatrix}\Lambda_{sr,i,1}^{-1} & \\ & \mathbf{0}\end{bmatrix}\begin{bmatrix}\mathbf{V}_{sr,i,1} & \mathbf{V}_{sr,i,2}\end{bmatrix}^{H} \\ &= \begin{bmatrix}\mathbf{V}_{sr,i,1} & \mathbf{V}_{sr,i,2}\end{bmatrix} \\ &\times\begin{bmatrix}\Lambda_{sr,i,1}^{-1}\mathbf{U}_{sr,i,1}^{H}\hat{\mathbf{U}}_{sr,i,2}\tilde{\Sigma}_{sr,i,1}\hat{\mathbf{U}}_{sr,i,1}\mathbf{U}_{sr,i,1}\Lambda_{sr,i,1}^{-1} & \\ & \mathbf{0}\end{bmatrix} \\ &\times\begin{bmatrix}\mathbf{V}_{sr,i,1} & \mathbf{V}_{sr,i,2}\end{bmatrix}^{H}\end{aligned} \quad (16)$$

It can be verified that $\mathbf{U}_{sr,i,1}^{H}\hat{\mathbf{U}}_{sr,i,1}$ is a unitary matrix, because $\mathbf{U}_{sr,i}^{H}\hat{\mathbf{U}}_{sr,i}$ is unitary. Recall that if $\mathbf{A}$ and $\mathbf{B}$ are two positive semi-definite $M \times M$ matrices with eigenvalues $\mathrm{a}_i(\mathbf{A})$ and $\mathrm{a}_i(\mathbf{B})$, arranged in the descending order respectively, then

$$\sum_{i=1}^{M}\mathrm{a}_i(\mathbf{A})\mathrm{a}_{M+1-i}(\mathbf{B}) \le Tr(\mathbf{AB}) \le \sum_{i=1}^{M}\mathrm{a}_i(\mathbf{A})\mathrm{a}_i(\mathbf{B}) \quad (17)$$

Then using the second inequality in (17) and knowing that $\mathbf{H}_{sr,i}^{H}\mathbf{H}_{sr,i}^{+H}\mathbf{H}_{sr,i}^{+}\mathbf{H}_{sr,i}$ is a project matrix with eigenvalues being only 1 and 0, we have

$$\begin{aligned}Tr(\mathbf{Q}_i) &\ge Tr\left(\mathbf{H}_{sr,i}^{+}\mathbf{H}_{sr,i}\mathbf{Q}_i\mathbf{H}_{sr,i}^{H}\mathbf{H}_{sr,i}^{H+}\right) \\ &= Tr\left(\Lambda_{sr,i,1}^{-1}\mathbf{U}_{sr,i,1}^{H}\hat{\mathbf{U}}_{sr,i,2}\tilde{\Sigma}_{sr,i,1}\hat{\mathbf{U}}_{sr,i,1}\mathbf{U}_{sr,i,1}\Lambda_{sr,i,1}^{-1}\right)\end{aligned} \quad (18)$$

Using the first equality in (17) we can conclude that

$$Tr(\mathbf{Q}_i) \ge Tr\left(\tilde{\Sigma}_{sr,i,1}\Lambda_{sr,i,1}^{-2}\right) \quad (19)$$

Therefore, the structure of the optimal transmit covariance matrix in the desired link is given by

$$\mathbf{Q}_{opt,i} = \begin{bmatrix}\mathbf{V}_{sr,i,1} & \mathbf{V}_{sr,i,2}\end{bmatrix}\begin{bmatrix}\tilde{\Sigma}_{sr,i,1}\Lambda_{sr,i,1}^{-2} & \\ & \mathbf{0}\end{bmatrix}\begin{bmatrix}\mathbf{V}_{sr,i,1} & \mathbf{V}_{sr,i,2}\end{bmatrix}^{H} \quad (20)$$

which satisfies

$$\mathbf{H}_{sr,i}\mathbf{Q}_{opt,i}\mathbf{H}_{sr,i}^{H} = \mathbf{U}_{sr,i}\begin{bmatrix}\tilde{\Sigma}_{sr,i,1} & \\ & \mathbf{0}\end{bmatrix}\mathbf{U}_{sr,i}^{H} \quad (21)$$

and the proposition is proved.

3-3- Problem reformulation

In the previous section we proved that the structure of the optimal amplification matrix in SU i and transmit covariance matrix in the SU TX can be expressed as

$$\mathbf{A}_{i,opt} = \mathbf{V}_{rd,i}\Lambda_{\mathbf{A}_i}\mathbf{U}_{sr,i}^{H}, \quad \mathbf{Q}_i = \mathbf{V}_{sr,i}\Lambda_{\mathbf{Q}_i}\mathbf{V}_{sr,i}^{H} \quad (22)$$

where $\mathbf{H}_{sr,i} = \mathbf{U}_{sr,i}\Lambda_{sr,i}\mathbf{V}_{sr,i}^{H}$ and $\mathbf{H}_{rd,i} = \mathbf{U}_{rd,i}\Lambda_{rd,i}\mathbf{V}_{rd,i}^{H}$. Recall that the received signal in the SU RX, due to the cooperation of SU i, is given by

$$\mathbf{y}_d = \mathbf{H}_{rd,i}\mathbf{A}_i\mathbf{H}_{sr,i}\mathbf{x}_{s,i} + \mathbf{H}_{rd,i}\mathbf{A}_i\mathbf{n}_{r,i} + \mathbf{n}_d \quad (23)$$

Using (22), $\mathbf{y}_d$ in (23) can be rewritten as

$$\mathbf{y}_d = \mathbf{U}_{rd,i}\Lambda_{rd,i}\Lambda_{\mathbf{A}_i}\Lambda_{sr,i}\mathbf{V}_{sr,i}^{H}\mathbf{x}_{s,i} + \mathbf{U}_{rd,i}\Lambda_{rd,i}\Lambda_{\mathbf{A}_i}\mathbf{U}_{sr,i}^{H}\mathbf{n}_{r,i} + \mathbf{n}_d \quad (24)$$

Suppose that $\tilde{\mathbf{y}}_d = \mathbf{U}_{rd,i}^{H}\mathbf{y}_d$, $\tilde{\mathbf{x}}_{s,i} = \mathbf{V}_{sr,i}^{H}\mathbf{x}_{s,i}$, $\tilde{\mathbf{n}}_{r,i} = \mathbf{U}_{sr,i}^{H}\mathbf{n}_{r,i}$ and $\tilde{\mathbf{n}}_d = \mathbf{U}_{rd,i}^{H}\mathbf{n}_d$. Then,

$$\tilde{\mathbf{y}}_d = \Lambda_{rd,i}\Lambda_{\mathbf{A}_i}\Lambda_{sr,i}\tilde{\mathbf{x}}_{s,i} + \Lambda_{rd,i}\Lambda_{\mathbf{A}_i}\tilde{\mathbf{n}}_{r,i} + \tilde{\mathbf{n}}_d \quad (25)$$

Clearly, the relay channel between the SU TX and SU RX has been decomposed into a set of parallel SISO sub channels. Therefore, the achievable data rate in the desired link as result of the cooperation of SU i can be expressed as

$$R_i = \log_2\left|\mathbf{I}_{N_d} + \Lambda_{rd,i}^2\Lambda_{\mathbf{A}_i}^2\Lambda_{sr,i}^2\Lambda_{\mathbf{Q}_i}\left(\pm_r^2\Lambda_{rd,i}^2\Lambda_{\mathbf{A}_i}^2 + \pm_d^2\mathbf{I}_{N_d}\right)^{-1}\right| \quad (26)$$

Suppose that the eigenvalue decomposition of $\mathbf{H}_{s,p,n}^{H}\mathbf{H}_{s,p,n}$ and $\mathbf{H}_{i,p,n}^{H}\mathbf{H}_{i,p,n}$ is

$$\begin{aligned}\mathbf{H}_{s,p,n}^{H}\mathbf{H}_{s,p,n} &= \mathbf{U}_{s,p,n}\Lambda_{s,p,n}\mathbf{U}_{s,p,n}^{H} \\ \mathbf{H}_{i,p,n}^{H}\mathbf{H}_{i,p,n} &= \mathbf{U}_{i,p,n}\Lambda_{i,p,n}\mathbf{U}_{i,p,n}^{H}\end{aligned} \quad (27)$$

For all $n \in S_{PU}$. We further assume that

$$\begin{aligned}&\Lambda_{\mathbf{A}_i}^2 = \mathrm{diag}\left\{a_{1,i},\ldots,a_{N_r,i}\right\}, \quad \Lambda_{sr,i}^2 = \mathrm{diag}\left\{b_{1,i},\ldots,b_{N_r,i}\right\} \\ &\Lambda_{rd,i}^2 = \mathrm{diag}\left\{c_{1,i},\ldots,c_{N_r,i}\right\}, \quad \Lambda_{s,p,n} = \mathrm{diag}\left\{d_{1,n},\ldots,d_{N_s,n}\right\} \\ &\Lambda_{i,p,n} = \mathrm{diag}\left\{e_{1,i,n},\ldots,e_{N_r,i,n}\right\}, \quad \Lambda_{\mathbf{Q},i} = \mathrm{diag}\left\{q_{1,i},\ldots,q_{N_s,i}\right\}\end{aligned} \quad (28)$$

Then, using (28), (26) can be rewritten as

$$R_i = \sum_{k=1}^{N_r}\log_2\left(1 + \frac{a_{k,i}b_{k,i}c_{k,i}q_{k,i}}{\pm_r^2 a_{k,i}c_{k,i} + \pm_d^2}\right) \quad (29)$$

Moreover, the transmit power constraint of the SU TX and SU i will become

$$Tr(\mathbf{Q}_i) \le P_T \Rightarrow \sum_{k=1}^{N_s}q_{k,i} \le P_T \quad (30)$$

$$\begin{aligned}&Tr\left[\mathbf{A}_i\left(\pm_r^2\mathbf{I}_{N_r} + \mathbf{H}_{sr,i}\mathbf{Q}_i\mathbf{H}_{sr,i}^{H}\right)\mathbf{A}_i^{H}\right] \\ &= Tr\left(\pm_r^2\Lambda_{\mathbf{A}_i}^2 + \Lambda_{sr,i}^2\Lambda_{\mathbf{Q}_i}\Lambda_{\mathbf{A}_i}^2\right) \le P_R \Rightarrow \\ &\sum_{k=1}^{N_r}a_{k,i}\left(\pm_r^2 + b_{k,i}q_{k,i}\right) \le P_R\end{aligned} \quad (31)$$

The interference constraint on PUs, due to transmission by the SU TX, can be written as

$$Tr\left(\mathbf{H}_{s,p,n}\mathbf{Q}_i\mathbf{H}_{s,p,n}^{H}\right) = Tr\left(\mathbf{V}_{sr,i}^{H}\mathbf{U}_{s,p,n}\Lambda_{s,p,n}\mathbf{U}_{s,p,n}^{H}\mathbf{V}_{sr,i}\Lambda_{\mathbf{Q}_i}\right) \quad (32)$$

for all $n \in S_{PU}$. Let $\mathbf{M}_{i,n} = \mathbf{V}_{sr,i}^{H}\mathbf{U}_{s,p,n}$. Then, (32) can be expressed as

$$Tr\left(\mathbf{M}_{i,n}\Lambda_{s,p,n}\mathbf{M}_{i,n}^{H}\Lambda_{\mathbf{Q}_i}\right) = \sum_{k=1}^{N_s}\left(\sum_{l=1}^{N_s}\left|m_{i,k,l,n}\right|^2 d_{l,n}\right)q_{k,i} \quad (33)$$

where $m_{i,k,l,n}$ denotes the element of k-th row and l-th column of matrix $\mathbf{M}_{i,n}$. Let $f_{k,i,n} = \sum_{l=1}^{N_s}\left|m_{i,k,l,n}\right|^2 d_{l,n}$. Therefore, the interference constraint on PUs, due to transmitting by SU TX, is expressed by

$$Tr\left(\mathbf{H}_{s,p.n}\mathbf{Q}_i\mathbf{H}_{s,p,n}^{H}\right) = \sum_{k=1}^{N_s} f_{k,i,n}q_{k,i} \le P_{I,1} \quad (34)$$

The interference constraint on PUs, due to the cooperation of SU i with SU TX, is written by

$$\begin{aligned} & Tr\left\{\mathbf{H}_{i,p,n}\mathbf{A}_i\left(\pm_r^2\mathbf{I}_{N_r} + \mathbf{H}_{sr,i}\mathbf{Q}_i\mathbf{H}_{sr,i}^{H}\right)\mathbf{A}_i^H\mathbf{H}_{i,p,n}^{H}\right\} \\ & = Tr\left(\mathbf{V}_{rd,i}^{H}\mathbf{U}_{i,p,n}\Lambda_{i,p,n}\mathbf{U}_{i,p,n}^{H}\mathbf{V}_{rd,i}\Lambda_{\mathbf{A}_i}\right. \\ & \left.\times\left(\pm_r^2\mathbf{I}_{N_r} + \Lambda_{sr,i}\Lambda_{\mathbf{Q}_i}\Lambda_{sr,i}\right)\mathbf{U}_{sr,i}\Lambda_{\mathbf{A}_i}\right) \le P_{I,2} \end{aligned} \quad (35)$$

for all $i \in S_R$ and $n \in S_{PU}$. Let $\mathbf{S}_{i,n} = \mathbf{V}_{rd,i}^{H}\mathbf{U}_{i,p,n}$. The element of k-th row and l-th column of $\mathbf{S}_{i,n}$ is denoted by $s_{i,k,l,n}$. Hence, it can be shown that (35) can be rewritten as

$$\sum_{k=1}^{N_r}\left(\pm_r^2 + b_{k,i}q_{k,i}\right)a_{k,i}\left(\sum_{l=1}^{N_r}\left|s_{i,k,l,n}\right|^2 e_{l,i,n}\right) \le P_{I,2} \quad (36)$$

Let $g_{k,i,n} = \sum_{l=1}^{N_r}\left|s_{i,k,l,n}\right|^2 e_{l,i,n}$. Thus, the interference constraint on PUs, due to the cooperation of the selected SU in relaying the signals of the SU TX is stated as

$$\sum_{k=1}^{N_r}\left(\pm_r^2 + b_{k,i}q_{k,i}\right)a_{k,i}g_{k,i,n} \le P_{I,2} \quad (37)$$

Let $\mathbf{a}_i = \left[a_{1,i},\ldots,a_{N_r,i}\right]$ and $\mathbf{q}_i = \left[q_{1,i},\ldots,q_{N_s,i}\right]$

Finally, the problem (4) can be expressed according to the following

$$\begin{aligned} & \mathbf{q}_i^*,\mathbf{a}_i^* = \arg\max_{\mathbf{q}_i,\mathbf{a}_i} \sum_{k=1}^{N_r}\log_2\left(1+\frac{a_{k,i}b_{k,i}c_{k,i}q_{k,i}}{\pm_r^2 a_{k,i}c_{k,i} + \pm_d^2}\right) \\ & i = \arg\max_i \sum_{k=1}^{N_r}\log_2\left(1+\frac{a_{k,i}^*b_{k,i}c_{k,i}q_{k,i}^*}{\pm_r^2 a_{k,i}^*c_{k,i} + \pm_d^2}\right) \\ & \text{s. t.} \quad \sum_{k=1}^{N_s} q_{k,i} \le P_T \\ & \sum_{k=1}^{N_r} a_{k,i}\left(\pm_r^2 + b_{k,i}q_{k,i}\right) \le P_R \\ & \sum_{k=1}^{N_s} f_{k,i,n}q_{k,i} \le P_{I,1}, \quad \forall n \in S_{PU} \\ & \sum_{k=1}^{N_r} g_{k,i,n}a_{k,i}\left(\pm_r^2 + b_{k,i}q_{k,i}\right) \le P_{I,2}, \quad \forall n \in S_{PU} \end{aligned} \quad (38)$$

Let $h_{k,i} = a_{k,i}\left(\pm_r^2 + b_{k,i}q_{k,i}\right)$. By some simple derivations, the problem in (38) is equivalent to

$$\begin{aligned} & \mathbf{q}_i^*,\mathbf{h}_i^* = \arg\max_{\mathbf{q}_i,\mathbf{h}_i} \sum_{k=1}^{N_r}\log_2\frac{\left(1+\frac{c_{k,i}h_{k,i}}{\pm_d^2}\right)\left(1+\frac{b_{k,i}q_{k,i}}{\pm_r^2}\right)}{1+\frac{c_{k,i}h_{k,i}}{\pm_d^2}+\frac{b_{k,i}q_{k,i}}{\pm_r^2}} \\ & i = \arg\max_i \sum_{k=1}^{N_r}\log_2\frac{\left(1+\frac{c_{k,i}h_{k,i}^*}{\pm_d^2}\right)\left(1+\frac{b_{k,i}q_{k,i}^*}{\pm_r^2}\right)}{1+\frac{c_{k,i}h_{k,i}^*}{\pm_d^2}+\frac{b_{k,i}q_{k,i}^*}{\pm_r^2}} \\ & \text{s. t.} \quad \sum_{k=1}^{N_s} q_{k,i} \le P_T \\ & \sum_{k=1}^{N_r} h_{k,i} \le P_R \\ & \sum_{k=1}^{N_s} f_{k,i,n}q_{k,i} \le P_{I,1}, \quad \forall n \in S_{PU} \\ & \sum_{k=1}^{N_r} g_{k,i,n}h_{k,i} \le P_{I,2}, \quad \forall n \in S_{PU} \end{aligned} \quad (39)$$

4. Optimization Algorithm

In this section, we develop approaches for joint relay selection and power allocation in cooperative cognitive radio networks. At first, we provide an optimal approach and then develop a low-complexity suboptimal approach.

4-1- Optimal approach

Using the Lagrange multipliers method [26] the Lagrange function for (39) is given by

$$\begin{aligned} & L\left(\mathbf{h}_i,\mathbf{q}_i,\mathsf{a}_1,\mathsf{a}_2,\mathsf{a}_{3,n},\mathsf{a}_{4,n}\right) = \\ & -\sum_{k=1}^{N_r}\log_2\frac{\left(1+\frac{c_{k,i}h_{k,i}}{\pm_d^2}\right)\left(1+\frac{b_{k,i}q_{k,i}}{\pm_r^2}\right)}{1+\frac{c_{k,i}h_{k,i}}{\pm_d^2}+\frac{b_{k,i}q_{k,i}}{\pm_r^2}} \\ & +\mathsf{a}_1\left(\sum_{k=1}^{N_s}q_{k,i} - P_T\right) + \mathsf{a}_2\left(\sum_{k=1}^{N_r}h_{k,i} - P_R\right) + \\ & \sum_{n=1}^{N_{PU}}\mathsf{a}_{3,n}\left(\sum_{k=1}^{N_r}f_{k,i,n}q_{k,i} - P_{I,1}\right) \\ & +\sum_{n=1}^{N_{PU}}\mathsf{a}_{4,n}\left(\sum_{k=1}^{N_r}g_{k,i,n}h_{k,i} - P_{I,2}\right) \end{aligned} \quad (40)$$

where a_1, a_2, $\mathsf{a}_{3,n}$ and $\mathsf{a}_{4,n}$ are the Lagrange multipliers, $\forall n \in S_{PU}$. According to the KKT conditions, we have

$$\lambda_1 \geq 0,\ \lambda_2 \geq 0,\ \lambda_{3,n} \geq 0,\ \lambda_{4,n} \geq 0,$$
$$q_{l,i} \geq 0,\ h_{k,i} \geq 0,\ l = 1,\ldots,N_s,\ k = 1,\ldots,N_r$$
$$\lambda_1\left(\sum_{k=1}^{N_s} q_{k,i} - P_T\right) = 0$$
$$\lambda_2\left(\sum_{k=1}^{N_s} h_{k,i} - P_R\right) = 0$$
$$\lambda_{3,n}\left(\sum_{k=1}^{N_s} f_{k,i,n} q_{k,i} - P_{I,1}\right) = 0$$
$$\lambda_{4,n}\left(\sum_{k=1}^{N_r} g_{k,i,n} h_{k,i} - P_{I,2}\right) = 0$$
$$\frac{\partial L}{\partial q_{k,i}} = 0, \quad k = 1,\ldots,N_s$$
$$\frac{\partial L}{\partial h_{k,i}} = 0, \quad k = 1,\ldots,N_r \tag{41}$$

For all $n \in S_{PU}$ and $i \in S_R$. It can be shown that $h_{k,i}$ and $q_{k,i}$ can be obtained using the following equations

$$q_{k,i} = \frac{\sigma_r^2}{2b_{k,i}}\left[\sqrt{\frac{c_{k,i}^2}{\sigma_d^4}h_{k,i}^2 - \frac{4b_{k,i}c_{k,i}}{\left(\lambda_1 + f_{k,i,n}\lambda_3\right)\sigma_r^2\sigma_d^2 \ln 2}h_{k,i}} - \left(2 + \frac{c_{k,i}}{\sigma_d^2}h_{k,i}\right)\right]^+ \tag{42}$$

$$h_{k,i} = \frac{\sigma_d^2}{2c_{k,i}}\left[\sqrt{\frac{b_{k,i}^2}{\sigma_r^4}q_k^2 - \frac{4b_{k,i}c_{k,i}}{\left(\lambda_2 + g_{k,i,n}\lambda_4\right)\sigma_r^2\sigma_d^2 \ln 2}q_k} - \left(2 + \frac{b_{k,i}}{\sigma_r^2}q_{k,i}\right)\right]^+ \tag{43}$$

where $[.]^+ = \max(.,0)$. Using dual-domain and sub-gradient methods [27], we can further obtain λ_1, λ_2, $\lambda_{3,n}$ and $\lambda_{4,n}$ through iteration,

$$\lambda_1^{(m+1)} = \left[\lambda_1^{(m)} + \alpha^{(m)}\left(\sum_{k=1}^{N_s} q_{k,i}^{(m)} - P_T\right)\right]^+$$
$$\lambda_2^{(m+1)} = \left[\lambda_2^{(m)} + \alpha^{(m)}\left(\sum_{k=1}^{N_r} h_{k,i}^{(m)} - P_R\right)\right]^+$$
$$\lambda_{3,n}^{(m+1)} = \left[\lambda_{3,n}^{(m)} + \alpha^{(m)}\left(\sum_{k=1}^{N_r} f_{k,i,n} q_{k,i}^{(m)} - P_{I,1}\right)\right]^+, \forall n \in S_{PU} \tag{44}$$
$$\lambda_{4,n}^{(m+1)} = \left[\lambda_{4,n}^{(m)} + \alpha^{(m)}\left(\sum_{k=1}^{N_r} g_{k,i,n} h_{k,i}^{(m)} - P_{I,2}\right)\right]^+, \forall n \in S_{PU}$$

where m is the iteration index and $\alpha^{(m)}$ is a sequence of scalar step sizes. Once λ_1, λ_2, $\lambda_{3,n}$ and $\lambda_{4,n}$ are obtained, we can get the optimal power allocation matrices $\mathbf{Q}_i$ and $\mathbf{A}_i$ and the corresponding achievable data rate R_i when the i-th SU acts as the relay for the SU TX. Repeating the above procedures at all SUs, we then find the one with the maximum achievable data rate.

4-2- Low-complexity approach

The optimal approach performs joint opportunistic relay selection and power allocation and results in the maximum data rate. However, the optimal approach is with very high complexity. Here, we aim to develop an alternate low-complexity suboptimal approach for problem (39). At first, we assume that the available source power is distributed uniformly over the spatial modes, i.e. $q_i^{uni} = \frac{P_T}{N_s}$. Similar assumption applies for $h_{k,i}$ ($k = 1,\ldots,N_r$), i.e. $h_i^{uni} = \frac{P_R}{N_r}$. Also assume that the interference introduced to the PU by each spatial mode of SU TX is equal and hence the maximum allowable power that can be allocated to the k-th mode is $q_{k,i}^{\max} = \frac{P_{I,1}}{N_s f_{k,i}^{\max}}$, where $f_{k,i}^{\max} = \max_{n \in S_{PU}} f_{k,i,n}$. Therefore, the allocated power to the k-th mode in the SU TX, intended for SU i, is $q_{k,i}^* = \min\left\{q_i^{uni}, q_{k,i}^{\max}\right\}$ for $k = 1,\ldots,N_s$ and $\forall i \in S_R$. Similarly, we assume that the interference introduced to the PU by each spatial mode of SU i is equal. Therefore, it can be concluded that $h_{k,i}^{\max} = \frac{P_{I,2}}{N_r g_{k,i}^{\max}}$, where $g_{k,i}^{\max} = \max_{n \in S_{PU}} g_{k,i,n}$. Therefore, the power allocation in the SU i is given by $h_{k,i}^* = \min\left\{h_i^{uni}, h_{k,i}^{\max}\right\}$ for $k = 1,\ldots,N_r$ and $\forall i \in S_R$. Afterwards, the SU i is selected as the cooperative relay such that the following is maximized

$$i = \arg\max_i \sum_{k=1}^{N_r} \log_2 \frac{\left(1 + \frac{c_{k,i}h_{k,i}^*}{\sigma_d^2}\right)\left(1 + \frac{b_{k,i}q_{k,i}^*}{\sigma_r^2}\right)}{1 + \frac{c_{k,i}h_{k,i}^*}{\sigma_d^2} + \frac{b_{k,i}q_{k,i}^*}{\sigma_r^2}} \tag{45}$$

After determining the cooperative SU, we calculate the optimal transmit covariance matrix, $\mathbf{Q}_i$, and amplification matrix, $\mathbf{A}_i$, using the approach provided in the optimal approach subsection. As we can see from the simulation results, this approach is almost as good as the optimal approach. However, it is with much lower complexity.

5. Outage Analysis

In order to analyze the outage behaviour of the proposed system, we consider the scenario where the PU transmitters, $\text{PU TX}_1,\ldots,\text{PU TX}_{N_{PU}}$, randomly communicate with their respective receivers, $\text{PU RX}_1,\ldots,\text{PU RX}_{N_{PU}}$. The interval between two transmissions of PUs and the duration of one PU transmission are assumed being random and obeying Exponential distribution with two parameters μ and ν, respectively. According to queuing theory, the probability of the absence of the PUs, $P\left(A\right)$, and the probability of the presence of the PUs, $P\left(\bar{A}\right)$, can be expressed respectively as $P\left(A\right)=\left(\sum_{n=0}^{N_{PU}}\frac{N_{PU}\,!}{\left(N_{PU}-n\right)!}\left(\frac{-}{2}\right)^{n}\right)^{-1}=\sim$ and $P\left(\bar{A}\right)=1-\sim$. In order to facilitate the analysis of outage, we modify the system model as explained below. First of all, we assume that the transmit signal at the SU TX is white and thereby $\mathbf{Q}=\circ\mathbf{I}_{N_s}$, where $\mathbf{I}_{N_s}$ represents the $N_s \times N_s$ identity matrix and $\circ N_s$ is the transmit power of the SU TX. Moreover, the cooperation strategy of the selected SU is assumed to be Decode-and-Forward (DF) strategy. This strategy switch is intended for some reasons, which among them is to obtain a lower bound for the outage capacity of the desired MIMO link. Meanwhile, this assumption facilitates the outage probability analysis, as will be shown below.

In the first time-slot, the spectrum sensing is used to detect whether the PUs are absent. When the PUs are absent, SU TX transmits data to SU RX directly. When the PUs are present, the transmit power of SU TX, $\circ N_s$, should be limited. However, if $\circ N_s$ is too low, the data from SU TX cannot reach SU RX. Thus, we use cooperative relaying to transmit signal from SU TX to SU RX through the best relay which is selected out of available SUs. In the sequel, we derive the approximate outage probabilities of the desired SU link, when the PUs are present and when no PUs transmit signals or in other words, the PUs are absent.

5-1- Absence of PUs

We firstly assume that no PU link is transmitting signal. Hence, the SU TX communicates directly with the SU RX and the received signal in the SU RX can be written as

$$\mathbf{y}_d=\mathbf{H}_{sd}\mathbf{x}_s+\mathbf{n}_d \qquad (46)$$

Based on the assumptions expressed at the beginning of this section, the achievable data rates of the desired link using the direct channel is given by

$$R^{D}=\log_2\left|\mathbf{I}_{N_d}+\frac{\circ}{\pm_d^2}\mathbf{H}_{sd}\mathbf{H}_{sd}^{H}\right| \qquad (47)$$

where $\mathbf{H}_{sd}\in\mathbb{C}^{N_d\times N_s}$ represents the direct channel in the desired link. It is obvious that the achievable data rates in the desired link, R^D, is a random variable which depends on the random nature of $\mathbf{H}_{sd}$. In a full-rank system, (47) can be simplified by using singular value decomposition (SVD) as

$$R^{D}=\sum_{m=1}^{N_d}\log_2\left(1+\frac{\circ}{\pm_d^2}\text{a}_{sd,m}\right) \qquad (48)$$

where $\text{a}_{sd,m}$, $i=1,\ldots,N_d$ are the non-negative eigenvalues of the channel covariance matrix $\mathbf{H}_{sd}\mathbf{H}_{sd}^{H}$. The joint pdf of $\text{a}_{sd,m}$, $i=1,\ldots,N_d$ is given by [11]

$$p\left(\text{a}_{sd,1},\ldots,\text{a}_{sd,N_d}\right)=\left(N_d\,!K_{N_d,N_s}\right)^{-1}\left(\prod_{m=1}^{N_d}\text{a}_{sd,m}^{N_r-N_d}\right) \times\left(\prod_{m<n}\left(\text{a}_{sd,m}-\text{a}_{sd,n}\right)^2\right)\exp\left(-\sum_{m=1}^{N_d}\text{a}_{sd,m}\right) \qquad (49)$$

where K_{N_d,N_s} is a normalizing factor. To ensure QoS for the desired link, it needs to support a minimum rate. When than the instantaneous achievable data rate is less than the minimum rate, $R_{\min}$, an outage event occurs. In quasi-static fading, since the fading coefficients are constant over the whole frame, we cannot average them with an ergodic measure. In such an event, Shannon capacity does not exist in the ergodic sense [28-30]. The probability of such an event is normally referred to as outage probability. As described in [31], the distribution of the random achievable data rate can be viewed as Gaussian when the number of transmit and/or receive antennas goes to infinity. It is also a very good approximation for even small N_d and N_s, e.g. $N_s=N_d=2$ [24]. As such, for a sufficiently large N_d and N_s, the achievable data rate of the desired link is approximated as [31]

$$R^{D}\to N\left(N_d\log_2\left(1+\circ\right),\frac{N_d\circ^2}{\left(\ln 2\right)^2N_s\left(1+\circ\right)^2}\right) \qquad (50)$$

Then, we proceed by considering the distribution of the achievable data rate in the desired link as Guassian with the pdf given in (50). Consequently, it can be shown that the outage probability of the desired link in the absence of the PUs can be written as

$$\begin{aligned}P_{out}^{D}&=P\left(R^{D}<R_{\min}\right)\\&=Q\left(\frac{N_d\log_2\left(1+\circ\right)-R_{\min}}{\sqrt{\frac{N_d\circ^2}{\left(\ln 2\right)^2N_r\left(1+\circ\right)^2}}}\right)\end{aligned} \qquad (51)$$

where $Q(.)$ denotes the Q-function.

5-2- Presence of PUs

As described in the previous section, when PUs transmit signals, the direct communication in the desired link must be avoided and the cooperation of the best SU is employed instead. The received signal in the SU RX using the cooperation of *i*-th SU can be expressed as

$$\mathbf{y}_d=\mathbf{H}_{rd,i}\mathbf{x}_{s,i}+\mathbf{n}_d \qquad (52)$$

Thus, the achievable data rates of the desired link is given by

$$R_i^{C}=\frac{1}{2}\log_2\left|\mathbf{I}_{N_d}+\frac{\circ}{\pm_d^2}\mathbf{H}_{rd,i}\mathbf{H}_{rd,i}^{H}\right| \qquad (53)$$

It can be concluded that for the case of present PUs, the achievable data rates in the desired link, R_i^C, can be expressed as

$$R_i^C = \frac{1}{2}\sum_{m=1}^{N_d} \log_2\left(1 + \frac{\circ}{\pm_d^2} \text{a}_{rd,i,m}\right) \tag{54}$$

where $\text{a}_{rd,i,m}$, $m = 1,\dots,N_d$ are the non-negative eigenvalues of the channel covariance matrix $\mathbf{H}_{rd,i}\mathbf{H}_{rd,i}^H$. The joint pdf of $\text{a}_{rd,i,m}$, $m = 1,\dots,N_d$ is given by [11]

$$p\left(\text{a}_{rd,i,1},\dots,\text{a}_{rd,i,N_d}\right) = \left(N_d!K_{N_d,N_r}\right)^{-1} \exp\left(-\sum_{m=1}^{N_d} \text{a}_{rd,i,m}\right) \times \left(\prod_{m=1}^{N_d} \text{a}_{rd,i,m}^{N_r - N_d}\right)\left(\prod_{m<n}\left(\text{a}_{rd,i,m} - \text{a}_{rd,i,n}\right)^2\right) \tag{55}$$

where K_{N_d,N_r} is a normalizing factor. Once again and similar to the previous discussions, the achievable data rate of the desired link is approximated as [24]

$$R_i^C \to N\left(N_d \log_2(1+\circ), \frac{N_d \circ^2}{4(\ln 2)^2 N_r (1+\circ)^2}\right) \tag{56}$$

Note that the coefficient $1/4$ in the variance of the pdf in (56) is due to the multiplication of $1/2$ in (54). Therefore, the outage probability of the desired link in the presence of the PUs can be written as

$$\begin{aligned} P_{out}^{C,i} &= P\left(R_i^C < R_{\min}\right) \\ &= Q\left(\frac{N_d \log_2(1+\circ) - R_{\min}}{\sqrt{\frac{N_d \circ^2}{4(\ln 2)^2 N_r (1+\circ)^2}}}\right) \end{aligned} \tag{57}$$

5-3- The outage probability

In this subsection, the outage probability of the proposed Cognitive Cooperative communication protocol based on Beam forming (CCB) is obtained. However, in the case that the DF cooperation strategy is employed and the PUs are present, another possible case in the proposed protocol is when no SU can decode the signal from SU TX. This may be due to detrimental effects of fading and path loss in the link from the SU TX to SUs. In this case, the SU TX indispensably transmits data to SU TX directly with limited power $\circ' N_s$ in order not to disturb the PUs. Assume that Δ_u is a non-empty sub-set of the N_{SU} secondary users who can decode the data of SU TX, i.e. $\Delta_u \subseteq S_R$, and $\bar{\Delta}_u$ is the complementary set of Δ_u . Suppose that ¤ is a null set. Then, the probability of existing no SU to decode the data of SU TX , $P_{out}^{¤}$, can be written as

$$P_{out}^{¤} = P\left(\Delta = ¤\right) = \prod_{m=1}^{N_{SU}} P_{out}^{R,m} \tag{58}$$

and $P_{out}^{R,m}$ (where $m \in S_R$) denotes the outage probability in the link from SU TX to the SUs in the first time-slot. Similar to previous subsections, a good approximate for $P_{out}^{R,m}$ can be obtained as

$$P_{out}^{R,m} = Q\left(\frac{N_r \log_2(1+\circ') - R_{\min}}{\sqrt{\frac{N_r \circ'^2}{(\ln 2)^2 N_s (1+\circ')^2}}}\right) \tag{59}$$

In the following theorem, we derive the outage probability of the desired SU link using the CCB.

Theorem 1: *The outage probability of the desired SU link using the proposed cognitive cooperative communication protocol based on beamforming is*

$$P_{out} = (1-\sim)\left(P_{out}^{¤} + \sum_{u=1}^{2^{N_{SU}}-1} P\left(\Delta = \Delta_u\right) P_{out}^{\Delta_u}\right) + \sim P_{out}^{D} \tag{60}$$

where $P_{out}^{\Delta_u}$ is the outage probability of the desired link in the presence of PUs and when the one SUs in the sub-set Δ_u is cooperating with desired link.

Proof. Consider the case that the PUs are present. Then, the probability of event $\{\Delta = \Delta_u\}$, i.e. there exist some SUs which can decode the signal from SU TX, can be written as

$$P\left(\Delta = \Delta_u\right) = \left(\prod_{m\in\Delta_u}\left(1 - P_{out}^{R,m}\right)\right)\left(\prod_{m\in\bar{\Delta}_u} P_{out}^{R,m}\right) = {}_u \tag{61}$$

The outage probability of the desired link in the presence of PUs and when the one SUs in the sub-set Δ_u is cooperating with desired link is given by

$$P_{out}^{\Delta_u} = \prod_{i\in\Delta_u} P_{out}^{C,i} \tag{62}$$

Then, the outage probability of the desired SU link in the presence of the PU signals can be written as

$$P_{out}^{\bar{A}} = P_{out}^{¤} + \sum_{u=1}^{2^{N_{SU}}-1} P\left(\Delta = \Delta_u\right) P_{out}^{\Delta_u} \tag{63}$$

Finally, it can be concluded that the outage probability of the desired link using the proposed cognitive cooperative communication protocol based on beam forming is given by

$$\begin{aligned} P_{out} &= P\left(\bar{A}\right) P_{out}^{\bar{A}} + P\left(A\right) P_{out}^{A} \\ &= (1-\sim)\left(P_{out}^{¤} + \sum_{u=1}^{2^{N_{SU}}-1} P\left(\Delta = \Delta_u\right) P_{out}^{\Delta_u}\right) + \sim P_{out}^{D} \end{aligned} \tag{64}$$

where P_{out}^{D} is the outage probability of the desired link, when the PUs are absent and is given in (51) and the proof is complete in this way.

6. Simulation Results

In this section, the performance of the proposed CCB protocol is evaluated using simulations. For better comprehending the merit of the proposed low complexity approach (LCA), we will also compare the proposed approach with the approaches using random cooperative SU selection with optimal power allocation matrices (transmit covariance matrix and amplification matrix), referred to as RS-OPA (Random SU-Optimal Power Allocation) and non-optimal power allocation, i.e., the amplification matrix of the randomly selected SU and the transmit covariance matrix are obtained as described in 4.2, respectively, which is referred to as RS-EPA (Random SU-Equal Power Allocation). All users are assumed to be equipped with the same number of antennas, denoted by *N*.

We set interference limits, $P_{I,1} = P_{I,2} = 0.1$ mW , otherwise stated. There exist 5 PU pairs in the system, otherwise stated. The elements of the channel matrices follow a Rayleigh distribution and are independent of each other. The path-loss exponent is 4, and the standard deviation of shadowing is 6 dB. The number of existing SUs in the system is 20, otherwise stated. The level of noise is assumed identical in the system and equal to 10^{-6} W/Hz.

The data rate in the desired SU link versus the maximum transmit power of SU TX for different number of antennas and various scenarios is shown in Fig. 2. The maximum transmit power of each SU *i*, for all $i \in S_R$, is $P_R = 0.7$ W . Using the Low Complexity Approach (LCA), 50% achievable data rate gain over the RS-OPA is obtained, when $N = 2$. Moreover, LCA leads to only 14% data rate degradation compared with OA, with much lower complexity. When P_T is small, the achievable data rate in the desired SU link increases rapidly with P_T. However, for large amounts of P_T , due to restrictions by the interference limits, the data rate is not sensitive to the P_T. As another observation, it can also be seen that the Random SU and Optimal Power Allocation scheme (RS-OPA) achieves a significant gain in the data rate over the Random SU and Non-Optimal (Equal) Power Allocation scheme (RS-EPA), especially when P_T is small.

The data rate of the desired SU link versus the maximum transmit power of the cooperating SU (P_R) is depicted in Fig. 3. The maximum transmit power of the SU TX is fixed at $P_T = 0.7$ W and the number of existing SUs in the secondary network, N_{SU}, is 20.

As shown in Fig. 4, the achievable data rate in the desired link grows with the number of existing SUs in the CR network. However, this growth saturates from a particular number of SUs which shows that the increasing the number of existing SUs will not necessarily result in the similar increase in the data rate of the desired link. Moreover, deploying larger number of antennas in users, i.e. larger *N*, compensates for the less maximum transmit power of SU TX and the cooperating relay. It must also be noted that the achievable data rate in the system is increased with the number of existing SUs due to multiuser diversity.

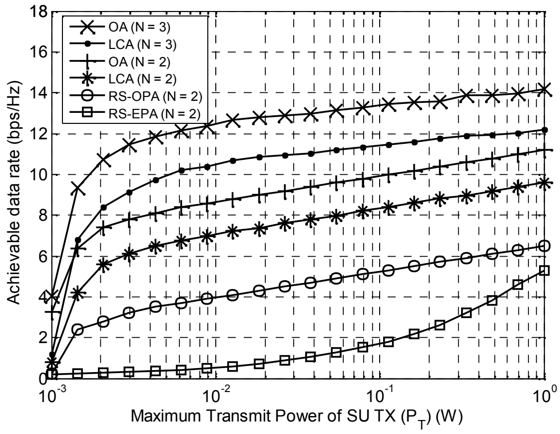

Fig. 2 Achievable data rate in the desired link versus the maximum transmit power of SU TX (P_T)

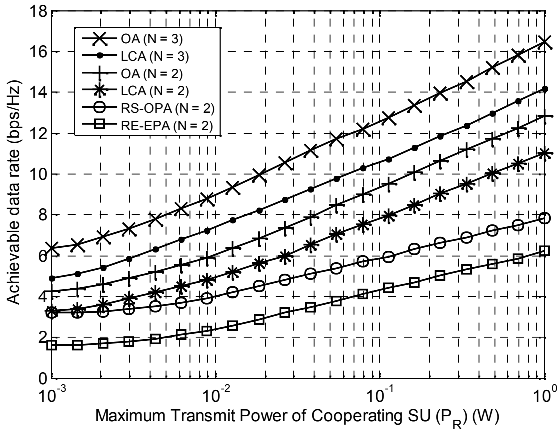

Fig. 3 Data rate in the desired link versus maximum transmit power of cooperating SU (P_R)

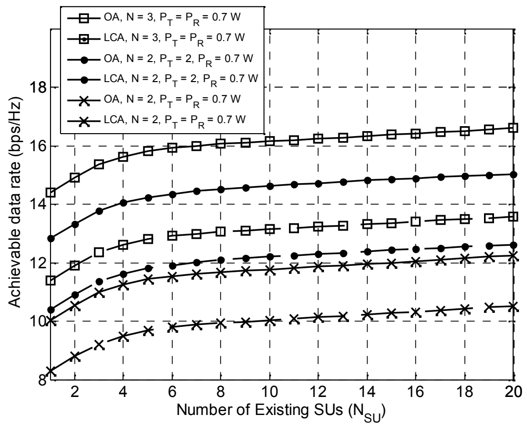

Fig. 4 Data rate in the desired link versus the number of existing SUs

7. Conclusions

In this work, an adaptive transmission protocol based on beam forming for underlay MIMO cognitive radio networks was proposed. It is assumed that when PUs are present, the direct transmission by PUs introduces intolerable interference on PUs. As a remedy, the cooperation of one of SUs was proposed to not only reduce the imposed interference on PUs, but also to maximize the data rates in the SU link. Based on the proposed Cognitive Cooperative communication protocol based on Beamforming (CCB), the joint problems of optimal power allocation and relay selection were solved in the optimal manner. However, due to high complexity of the optimal approach, a suboptimal approach with less complexity was further suggested. Finally, an outage probability analysis was provided to examine the performance of the proposed CCB protocol.

Appendix

Proof of Proposition 1.

It was shown in [25] that if the SU TX works in spatial multiplexing mode, i.e., the SU TX transmits independent data streams from different antennas, the amplification matrix of SU *i* can be written as

$$\mathbf{A}_i = \mathbf{V}_{rd,i} \Lambda_{\mathbf{A}_i} \mathbf{U}_{sr,i}^H \tag{65}$$

where $\Lambda_{\mathbf{A}_i}$ is a diagonal matrix. Therefore, $\mathbf{A}_i$ can be considered as a matched filter along the singular vectors of the channel matrices. In order to use the results of [25] for the case of non-white transmit data of the SU TX and equivalently the transmit covariance matrix is any arbitrary matrix $\mathbf{Q}_i$, we define the equivalent channel matrix $\tilde{\mathbf{H}}_{sr,i} = \mathbf{H}_{sr,i} \mathbf{Q}_i^{\frac{-1}{2}}$. Hence, by adopting the same method as in [25], for any given pair of $\mathbf{A}_i$ and $\tilde{\mathbf{Q}}_i$, there always exists another pair $\mathbf{A}_{i,opt}$ and $\tilde{\mathbf{Q}}_i$ that achieves better or equal data rate in the desired link. In this case, for the case of known $\mathbf{Q}_i$, (65) must be modified as

$$\mathbf{A}_{i,opt} = \mathbf{V}_{rd,i} \Lambda_{\mathbf{A}_i} \tilde{\mathbf{U}}_{sr,i}^H \tag{66}$$

where $\tilde{\mathbf{U}}_{sr,i}$ is obtained by eigenvalue decomposition of $\tilde{\mathbf{H}}_{sr,i} \tilde{\mathbf{H}}_{sr,i}^H$, i.e. $\tilde{\mathbf{H}}_{sr,i} \tilde{\mathbf{H}}_{sr,i}^H = \mathbf{H}_{sr,i} \tilde{\mathbf{Q}}_i \mathbf{H}_{sr,i}^H = \tilde{\mathbf{U}}_{sr,i} \tilde{\Sigma}_{sr,i} \tilde{\mathbf{U}}_{sr,i}^H$.

References

[1] S. Haykin, "Cognitive radio: Brain-empowered wireless communications," IEEE Journal on Selected Areas in Communications, vol. 23, pp. 201–220, Feburary 2005.

[2] Q. Zhao and B. M. Sadler, "A survey of dynamic spectrum access," IEEE Signal Process. Mag., vol. 24, no. 3, pp. 79–89, May 2007.

[3] V. Chakravarthy, X. Li, Z. Wu, M. A. Temple, F. Garber, R. Kannan, and A. Vasilakos, "Novel overlay/underlay cognitive radio waveforms using sd-smse framework to enhance spectrum efficiency-part I: Theoretical framework and analysis in AWGN channel," IEEE Trans. on Communications, vol. 57, no. 12, pp. 3794–3804, December 2009.

[4] R. Zhang, X. Kang, and Y. C. Liang, "Protecting primary users in cognitive radio networks: Peak or average interference power constraint?" in IEEE International Conference on Communications, Dresden, Germany, June 2009, pp. 1–5.

[5] S. Yan and X. Wang, "Power allocation for cognitive radio systems based on nonregenerative OFDM relay transmission," in International Conference on wireless communications, networking and mobile computing, Beijing, China, Sept. 2009, pp. 1–4.

[6] C. Luo, F. R. Yu, H. Ji, and V. C. Leung, "Distributed relay selection and power control in cognitive radio networks with cooperative transmission," in IEEE International Conference on Communications, Cape Town, South Africa, May 2010, pp. 1–5.

[7] J. Mietzner, L. Lampe, and R. Schober, "Distributed transmit power allocation for multichip cognitive radio systems," IEEE Trans. on Wireless Communications, vol. 8, no. 10, pp. 5187–5201, Oct. 2009.

[8] M. Xie, W. Zhang, and K.-K. Wong, "A geometric approach to improve spectrum efficiency for cognitive relay networks," IEEE Trans. On Wireless Communications, vol. 9, no. 1, pp. 268–281, Jan. 2010.

[9] C. Sun and K. B. Letaief, "User cooperation in heterogeneous cognitive radio network with interference reduction," in IEEE International Conference on Communications, Beijing, China, May 2008, pp. 3193–3197.

[10] M. A. Beigi and S. M. Razavizadeh, "Cooperative beam forming in cognitive radio networks," in IFIP Wireless Days, Paris, France, Dec. 2009, pp. 1–5.

[11] E. Telatar, "Capacity of multi-antenna Gaussian channels," Eur. Trans. Telecomm., vol. 10, pp. 585-598, Nov. 1999.

[12] G. J. Foschini and M. J. Gans, "On limits of wireless communications in a fading environment when using multiple antennas," Wireless Personal Commun., vol. 6, pp. 311-335, Mar. 1998.

[13] Y. Yu, W. Wang, C. Wang, F. Yan, and Y. Zhang, "Joint relay selection and power allocation with QoS support for cognitive radio networks," IEEE WCNC 2013, pp. 4516-4521.

[14] J. Kim, T. Q. Duong and X. Tran, "Performance analysis of cognitive spectrum-sharing single carrier systems with relay selection," IEEE Transactions on Signal Processing, vol. 60, no. 12, pp 6435-6449, 2012.

[15] M. G. Adian, and H. Aghaeinia, "Optimal resource allocation for opportunistic spectrum access in multiple-input multiple-output orthogonal frequency division multiplexing based cooperative cognitive radio networks," IET Signal Processing, vol. 7, no. 7, pp. 549-557, 2013.

[16] P. Ubaidulla and S. Aissa, "Optimal Relay Selection and Power Allocation for Cognitive Two-Way Relaying Networks," IEEE Wireless Communications Letters, vol. 1, no. 3, pp. 225-228, 2012.

[17] L. Li, X. Zhou, H. Xu, G. Y. Li, D. Wang and A. Soong, "Simplified relay selection and power allocation in cooperative cognitive radio systems," IEEE Transactions on Wireless Communications, vol. 10, no. 1, pp. 33-36, 2011.
[18] P. Li, S. Guo, W. Zhuang, and B. Ye, "On efficient resource allocation for cognitive and cooperative communications," IEEE Journal on Selected Areas in Communications, vol. 32, no. 2, pp. 264-273, 2013.
[19] M. G. Adian, H. Aghaeinia, and Y. Norouzi "Spectrum sharing and power allocation in multi input-multi-output multi-band underlay cognitive radio networks," IET Communications, vol. 7, no. 11, pp. 1140-1150, 2013.
[20] M. G. Adian, and H. Aghaeinia, "Spectrum sharing and power allocation in multiple-in multiple-out cognitive radio networks via pricing," IET Communications, vol. 6, no. 16, pp. 2621-2629, 2012.
[21] M. G. Adian, and H. Aghaeinia, "Resource allocation in MIMO-OFDM based cooperative cognitive radio networks," IEEE Transactions on Communications, doi: 10.1109/TCOMM.2014.2327063, 2014.
[22] M. G. Adian, and H. Aghaeinia, "Low complexity resource allocation in MIMO-OFDM-based cooperative cognitive radio networks," Transactions on Emerging Telecommunications Technology, doi: 10.1002/ett.2799, 2014.
[23] J. Liu, N. B. Shroff and H. D. Sherali, "Optimal Power Allocation in Multi-Relay MIMO Cooperative Networks: Theory and Algorithms," IEEE Journal on Selected Areas in Communications, vol. 30 , no. 2, 2012 , pp. 331 – 340.
[24] X. C.X. Wang, H. H. Chen and M. Guizani, "Cognitive radio network management," IEEE Vehicular Technology Magazine, vol. 3, no. 1, 2008, pp. 28-35.
[25] P. Gong, P. Xue, D. Park, and D. K. Kim "Optimum power allocation in a nonorthogonal amplify-and-forward relay-assisted network" IEEE Trans. Veh. Technol., vol. 60, no. 3, Mar. 2011, pp. 890–900.
[26] S. Boyd and L. Vandenberghe, Convex Optimization. Cambridge University Press, 2004.
[27] Y. Wei and R. Lui "Dual methods for nonconvex spectrum optimization of multicarrier systems" IEEE Trans. Commun., vol. 54, no. 7, July 2006, pp. 1310–1322.
[28] L. H. Ozarow, S. Shamai, and A. D. Wyner, "Information theoretic considerations for cellular mobile radio," IEEE Trans on Vehicular Technology, May 1994, vol. 43, pp. 359-378.
[29] R. Knopp and P. A. Humblet "On coding for block fading channels" IEEE Trans. on Information Theory, vol. 46, Jan. 2000, pp. 189-205.
[30] E. Biglieri, G. Caire, and G. Taricco "Limiting performance of block-fading channels with multiple antennas" IEEE Trans. on Information Theory, vol. 47, May 2001, pp. 1273-1289.
[31] B. M. Hochwald, T. L. Marzetta, and V. Tarokh "Multiple antenna channel-hardening and its implications for rate feedback and scheduling" IEEE Trans. on Information Theory, vol. 50, Sep. 2004, pp. 1893-1909.
[32] X. Tang and Y. Hua, "Optimal design of non-regenerative MIMO wireless relays,"IEEE Trans. on Wireless Communications, vol. 6, no. 4, 2007, pp. 1398-1407.

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