A Bias-reduced Solution for Target Localization with Distance-dependent Noises in Illuminator of Opportunity Passive Radar

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Abstract

A closed-form solution for target localization based on the realistic distance-dependent noises in illuminator of opportunity passive radar and the reduction method of the bias which exists in the two-stage weighted least squares (2SWLS) method is proposed. 2SWLS is a classic method for time-of-arrival (TOA) and frequency-of-arrival (FOA) localization problem and has a couple of improved solutions over the years. The 2SWLS and its improved solutions have great localization performances in their established location scenarios on the basis of two approximations that setting the noise to a constant and ignoring the high-order terms of TOA and FOA measurement noises. It is these two approximations that lead to a sub-optimal solution with bias. The bias of 2SWLS has a significant influence on the target localization in illuminator of opportunity passive radar that has lower measurement accuracy and higher noises than active radar. Therefore, this paper starts by taking into consideration of the realistic distance-dependent characteristics of TOA/-FOA noises and improving 2SWLS method. Then, the bias of the improved 2SWLS method is analyzed and a bias-reduced solution based on weighted least squares (WLS) is developed. Numerical simulations demonstrate that, compared to the existing improved solutions of the 2SWLS, the proposed method effectively reduces the bias and achieves higher localization accuracy.

Keywords: TOA; FOA; Target localization; Distance-dependent noises; Bias reduction; Illuminator of opportunity passive radar.

1. Introduction

The illuminator of opportunity passive radar has a unique advantage in terms of security and anti-stealth characteristics [1, 2]. Target localization with an illuminator of opportunity passive radar is a fundamental problem in the target reconnaissance field, which has received extensive attention in recent years [3-5].

Time-of-arrival (TOA) and frequency-of-arrival (FOA) measurements [6, 7] are mostly used in target localization methods. In Ho et al. [8], the classic two-stage weighted least squares (2SWLS) method is proposed. After transforming the measurement equations to a set of linear equations by introducing nuisance parameters and ignoring the second-order error, Ho et al. [8] applied weighted least squares (WLS) to obtain the estimated value of nuisance parameters, and get the estimates of position and velocity by using WLS again. Taking into account the accuracy drop of targets located on or near the axes, Xu et al. [9] proposed Turbo-2SWLS based on the anchor reference selection and coordinate rotation, thus achieving higher accuracy. However, the coordinate rotation process of Turbo-2SWSL required much higher computational complexity, especially as the number of transmitters and receivers increases. In Yang et al. [10], an auxiliary variable was introduced in the second stage of 2SWLS, and the new method is called Aux-2SWLS for short in this paper. By selecting the appropriate auxiliary variables, Aux-2SWLS could enhance the localization accuracy without increasing computational complexity. Amiri et al. [11, 12] proposed Ami-2SWLS method, which considered the realistic distance-dependent noises model [13], and achieved some improvement in most cases. However, it was demonstrated by our extensive calculations that there was a significant accuracy drop of velocity when the target was located on or near the axes.

In Ho et al. [8], Xu et al. [9], Yang et al. [10] and Amiri et al. [11], the high-order terms of noises were always ignored to reduce the computational complexity, but this ignores would bring an increasing bias in localization results as the TOA/FOA measurement noises got larger. Different from the situation of passive illuminators in Ho et al. [8], Xu et al. [9], Yang et al. [10] and Amiri et al. [11], the illuminators of passive radar is often non-cooperative and their signal waveform, frequency and phase cannot be obtained exactly most of the time. As a result, the TOA/FOA measurement noises are larger and signal-to-noise ratios are lower, and then the bias of localization is large enough to be taken into account to improve the localization accuracy [14]. Analyzed

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the bias in time-differences-of-arrival (TDOA) only and proposed a bias reduction method. However, aiming at the distance-dependent noises and the bias caused by higher TOA/FOA noises in this paper concerns, we will seek a more simple and effective method.

In this paper, the distance-dependent noises, which actually exist in the TOA/FOA measurements, are introduced first, and an improved method is derived based on Aux-2SWLS. Then, a bias-reduced solution of the improved method for TOA/FOA localization is proposed.

Finally, numerical simulations are carried out and the results show that this method could extremely attain the Cramer-Rao lower bound (CRLB) and lower the bias at higher noise levels.

2. An Improved 2SWLS With Distance-Dependent Noises

We consider the illuminator of opportunity passive radar system consisting of Nt illuminators expressed by I1, I2 ,…, I5 and Nr receivers expressed by R1, R2,…, R5 in Ns-dimensional space. Figure 1 shows a localization scenario of 5 illuminators and 5 receivers in 3-D space.

The position and velocity of the ith illuminator are denoted as $\mathbf{x}_{t,i} = [x_{t,i}^{(1)},\dots, x_{t,i}^{(Ns)}]^T$ and $\dot{\mathbf{x}}_{t,i} = [\dot{x}_{t,i}^{(1)}, \dots \dot{x}_{t,i}^{(Ns)}]^T$, and as $\mathbf{x}_{r,j} = [x_{r,j}^{(1)},\dots, x_{r,j}^{(Ns)}]^T$ and $\dot{\mathbf{x}}_{r,j} = [\dot{x}_{r,j}^{(1)}, \dots \dot{x}_{r,j}^{(Ns)}]^T$ denote the position and velocity of the *j*th receiver. The target position and velocity are represented by $\mathbf{x}_0 = [x_0^{(1)}, \dots, x_0^{(Ns)}]^T$ and $\dot{\mathbf{x}}_0 = [x_0^{(1)}, \dots \dot{x}_0^{(Ns)}]^T$.

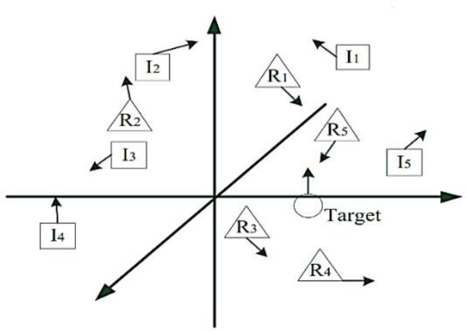


Fig. 1. Localization in 3-D space.

Each receiver can get Nt bistatic range (BR) and Nt bistatic range rate (BRR) measurements by Range-Doppler processing, that is, Nt TOA and Nt FOA data. Accordingly, there is a total of 2Nt Nr data that can be used for localization. Furthermore, the BR and BRR for the pair of the ith illuminator and the j th receiver can be computed from

$$d_{i,j} = d_i^t + d_j^r$$
$$\dot{d}_{i,j} = \dot{d}_i^t + \dot{d}_j^r \tag{1}$$

Where d_i^t and $\dot{d}_i^t$ are the range and range-rate between the ith illuminator and the target, and similarly, d_j^r and $\dot{d}_j^r$ the range and range-rate between the jth receiver and the target. They are given by

$$d_i^t = \|\mathbf{x}_0 - \mathbf{x}_{t,i}\|$$
$$\dot{d}_i^t = (\mathbf{x}_0 - \mathbf{x}_{t,i})^T(\dot{\mathbf{x}}_0 - \dot{\mathbf{x}}_{t,i})/d_i^t$$
$$d_j^r = \|\mathbf{x}_0 - \mathbf{x}_{r,j}\|$$
$$\dot{d}_j^r = (\mathbf{x}_0 - \mathbf{x}_{r,j})^T(\dot{\mathbf{x}}_0 - \dot{\mathbf{x}}_{r,j})/d_j^r \tag{2}$$

2.1 Distance-dependent noises

The measurement noises of BR and BRR, which are always considered to obey zero mean Gaussian distribution and independent of each other [10], are denoted as

$$q = [q_{1,1}, q_{1,2}, \dots , q_{N_t,N_r}]^T$$
$$\dot{q} = [\dot{q}_{1,1}, \dot{q}_{1,2}, \dots , \dot{q}_{N_t,N_r}]^T \tag{3}$$

Thus, the noisy measurements of BR and BRR can be expressed as

$$r_{i,j} = d_{i,j} + q_{i,j}$$
$$\dot{r}_{i,j} = \dot{d}_{i,j} + \dot{q}_{i,j} \tag{4}$$

The measurements of BR and BRR are always estimated by correlation processing in illuminator of opportunity passive radar. According to Stein [15], the standard deviations of BR and BRR noises are

$$\sigma_{i,j} = \frac{1}{f_{rms}} \frac{1}{\sqrt{BT(\frac{S}{N})}} \tag{5}$$

$$\dot{\sigma}_{i,j} = \frac{1}{T_{rms}} \frac{1}{\sqrt{BT(\frac{S}{N})}} \tag{6}$$

Where B is the noise bandwidth, T is the integration time, Trms and frms are the root mean square integration time and the root mean square radian frequency, S/N is the effective input signal-to-noise ratio. The four parameters B, T, Trms and frms depend on transmitted signal or receiver characteristics, while the parameter S/N depends on the distance and spatial geometry of illuminators, receivers and target. Hence S/N in (5) and (6) is the origin of the distance-dependent characteristics of BR and BRR noises. And then, from the bistatic radar equation [16], we have

$$\frac{S}{N} \;\alpha\; 1/(d_i^t d_j^r)^2 \tag{7}$$

That is to say,

$$\sigma_{i,j}^2 \;\alpha\; \dot{\sigma}_{i,j}^2 \;\alpha\; (d_i^t d_j^r)^2 \tag{8}$$

Where α is a symbol of the proportional relationship. So we can see that the noises are actually distance-dependent. In order to describe and analyze the problem conveniently, we assume that the illuminators have the same radiant power and the bistatic radar cross-section of the target is constant. This assumption will not affect the derivation and usability of the method to be proposed. Based on the above assumption, we denote the variances corresponding to $(d_i^t d_j^r)_{max}^2$ as σ_0^2 and $\dot{\sigma}_0^2$. Then, the variances for the pair of ith illuminator and the jth receiver can be denoted as

$$\begin{aligned} \sigma_{i,j}^2 &= \sigma_0^2 (d_i^t d_j^r)^2/(d_i^t d_j^r)_{max}^2 \\ \dot{\sigma}_{i,j}^2 &= \dot{\sigma}_0^2 (d_i^t d_j^r)^2/(d_i^t d_j^r)_{max}^2 \end{aligned} \tag{9}$$

And the BR and BRR covariance matrix is considered to be

$$\mathrm{Q} = diag\{\sigma_{1,1}^2, \sigma_{1,2}^2, \dots, \sigma_{N_t,N_r}^2, \dot{\sigma}_{1,1}^2, \dot{\sigma}_{1,2}^2, \dots, \dot{\sigma}_{N_t,N_r}^2\} \tag{10}$$

Where diag {*} denotes the diagonal matrix whose diagonal entries are the elements in braces.

Consequently, compared to the simple form of the covariance matrix without regard for the distance-dependent characteristics of noises,

$$\mathrm{Q}_{\text{simple}} = \mathrm{diag}\,\{\sigma_s^2, \sigma_s^2, \dots, \sigma_s^2, \dot{\sigma}_s^2, \dot{\sigma}_s^2, \dots, \dot{\sigma}_s^2\} \tag{11}$$

Where σ_s^2 and $\dot{\sigma}_s^2$ are constant variances of BR and BRR, covariance matrix in (10) is more realistic and complicated, and will be further researched.

2.2 An improved 2SWLS method

Aux-2SWLS is a creative and excellent improved version of the classic 2SWLS method, which enhances the localization performance to a great extent by introducing an auxiliary variable. Thus, we will derive an improved 2SWLS method with distance-dependent noises based on Aux-2SWLS method introduced in Yang et al. [10].

- Stage 1:

The estimate of $\boldsymbol{\theta}_0 = [\mathbf{x}_0^T, d_1^t, \dot{\mathbf{x}}_0^T, \dot{d}_1^t]^T$ is

$$\hat{\boldsymbol{\theta}}_a = (\mathbf{G}_a^T \mathbf{W}_a^{-1} \mathbf{G}_a)^{-1} \mathbf{G}_a^T \mathbf{W}_a^{-1} \mathbf{h}_a \tag{12}$$

Where

$$\begin{gathered} \mathbf{G}_a = [\mathbf{G}_{a,1}^T, \mathbf{G}_{a,2}^T, \dot{\mathbf{G}}_{a,1}^T, \dot{\mathbf{G}}_{a,2}^T]^T \\ [\mathbf{G}_a]_{j,:} = [2(\mathbf{x}_{r,j} - \mathbf{x}_{t,1})^T, -2r_{1,j}, \mathbf{0}] \\ [\mathbf{G}_a]_{k,:} = [2(\mathbf{x}_{t,i} - \mathbf{x}_{t,1})^T, 2(r_{i,j} - r_{1,j}), \mathbf{0}] \\ k = (i-2)N_r + j \\ i = 2, \dots, N_t, \quad j = 1, \dots, N_r \\ \mathbf{W}_a \approx \mathbf{T}_a \mathbf{Q} \mathbf{T}_a^T \\ \mathbf{T}_a = \begin{bmatrix} 2\mathbf{T}_{a,1}\boldsymbol{\Lambda}_a & \mathbf{0} \\ \dot{\mathbf{T}}_{a,1}\boldsymbol{\Lambda}_a & \mathbf{T}_{a,1}\Lambda_a \end{bmatrix} \\ \mathbf{T}_{a,1} = \begin{bmatrix} \mathbf{T}_{1,1} & \mathbf{0} \\ \mathbf{0} & \mathbf{T}_{1,2} \end{bmatrix} \\ \boldsymbol{\Lambda}_a = (\mathbf{I}_{N_t} + (\mathbf{e}_1 - \mathbf{1}_{N_t})\mathbf{e}_1^T) \otimes \mathbf{I}_{N_r} \\ \mathbf{T}_{1,1} = diag\{d_1^t - r_{1,1}, \dots, d_1^t - r_{1,N_r}\} \\ \mathbf{T}_{1,2} = diag\{r_{1,1} - r_{2,1} - d_1^t, \dots, r_{1,N_r} - r_{N_t,N_r} - d_1^t\} \\ \mathbf{h}_a = [\mathbf{h}_{a,1}^T, \mathbf{h}_{a,2}^T, \dot{\mathbf{h}}_{a,1}^T, \dot{\mathbf{h}}_{a,2}^T]^T, \\ [\mathbf{h}_a]_j = \mathbf{x}_{r,j}^T \mathbf{x}_{r,j} - \mathbf{x}_{t,1}^T \mathbf{x}_{t,1} - r_{1,j}^2, \\ [\mathbf{h}_a]_k = \mathbf{x}_{t,i}^T \mathbf{x}_{t,i} - \mathbf{x}_{t,1}^T \mathbf{x}_{t,1} - (r_{i,j} - r_{1,j})^2 \end{gathered} \tag{13}$$

Here, $\mathrm{I}_{\mathrm{N_t}}$ is an identity matrix of size $\mathrm{N_t}$, $1_{\mathrm{N_t}}$ is a $\mathrm{N_t}$-dimensional column vector whose elements are all 1, and e1 denotes a unit column vector proper in size, of which first element is 1 and other elements are 0. Besides, $[*]_{k,:}$ denotes the kth row of the matrix in square brackets, $[*]_k$ denotes the kth element of the vector in square brackets and $\otimes$ denotes the Kronecker product.

$\mathrm{W_a}$ is not known in practice as Ta and Q contain the true distances between illuminators, receivers and target. Further approximation is necessary to get $\mathrm{W_a}$. Normally, each $\mathrm{d_i^t}$ and $\dot{\mathrm{d}}_\mathrm{i}^\mathrm{t}$ is approximately at a similar level of magnitude as others, so that $\sigma_{i,j}$ and $\dot{\sigma}_{i,j}$ are close to σ_0 and $\dot{\sigma}_0$, respectively. Therefore, we have rough approximations

$$\begin{gathered} \mathbf{T}_a \approx d_0 \mathbf{I}_{2N_tN_r} \\ \mathbf{Q} \approx \sigma_0^2 diag\{1, 1, \dots, 1, \eta^2, \eta^2, \dots, \eta^2\} \end{gathered} \tag{14}$$

Where $\eta = \dot{\sigma}_0/\sigma_0$, which represents the ratio of standard deviation of BRR noises to BR noises. Admittedly, this approximation will be controversial, but it can be used to obtain an initial solution to estimate $\mathrm{W_a}$. Then, the initial $\hat{\theta}_a$ is computed from (12), and Ta, Q and $\mathrm{W_a}$ can be corrected by the computed $\hat{\theta}_a$. Finally, using (12) again, we can get the final estimate of $\hat{\theta}_a$.

- Stage 2:

Since d_1^t and $\dot{d}_1^t$ are dependent on x_0 and $\dot{x}_0$ respectively, we need to incorporate this relationship to get an improved estimate. In accordance with Yang et al. [10], the auxiliary variables $\alpha = [\hat{\theta}_a]_{1:N_s} - \gamma 1_{N_s}$ and $\dot{\alpha} = [\hat{\theta}_a]_{N_s+2:2N_s+1}$ are introduced, where can be any constant larger than several times or tens of times of $\hat{d}_1^t$, and we set

$$\boldsymbol{\theta}_b = \left(\begin{bmatrix} [\boldsymbol{\theta}]_{1:N_s} \\ [\boldsymbol{\theta}]_{1:N_s} \end{bmatrix} - \begin{bmatrix} \boldsymbol{\alpha} \\ \boldsymbol{\alpha} \end{bmatrix} \right) \odot \left(\boldsymbol{\theta} - \begin{bmatrix} \boldsymbol{\alpha} \\ \dot{\boldsymbol{\alpha}} \end{bmatrix} \right) \tag{15}$$

Where ʘ denotes element-wise product.
Then the estimate of (15) can be calculated by

$$\hat{\boldsymbol{\theta}}_b = (\mathbf{G}_b^T \mathbf{W}_b^{-1} \mathbf{G}_b)^{-1} \mathbf{G}_b^T \mathbf{W}_b^{-1} \mathbf{h}_b \tag{16}$$

where

$$\mathbf{G}_b = \mathbf{I}_2 \otimes [\mathbf{I}_{N_s}, \mathbf{1}_{N_s}]^T$$
$$\mathbf{W}_b \approx \mathbf{T}_b (\mathbf{G}_a^T \mathbf{W}_a^{-1} \mathbf{G}_a)^{-1} \mathbf{T}_b^T$$
$$\mathbf{T}_b = \begin{bmatrix} 2\mathbf{T}_{b,1} & \mathbf{0} \\ \mathbf{T}_{b,2} & \mathbf{T}_{b,1} \end{bmatrix}$$
$$\mathbf{T}_{b,1} = diag\{[\hat{\boldsymbol{\theta}}_a]_{1:N_s+1} - [\boldsymbol{\alpha}^T, 0]^T\} + \mathbf{e}_{N_s+1}[(\mathbf{x}_{t,1} - \boldsymbol{\alpha})^T, 0]$$
$$\mathbf{T}_{b,2} = diag\{[\hat{\boldsymbol{\theta}}_a]_{N_s+2:2N_s+2} - [\dot{\boldsymbol{\alpha}}^T, 0]^T\} + \mathbf{e}_{N_s+1}[(\dot{\mathbf{x}}_{t,1} - \dot{\boldsymbol{\alpha}})^T, 0]$$

$$\mathbf{h}_b = \begin{bmatrix} ([\hat{\boldsymbol{\theta}}_a]_{1:N_s} - \boldsymbol{\alpha}) \odot ([\hat{\boldsymbol{\theta}}_a]_{1:N_s} - \boldsymbol{\alpha}) \\ [\hat{\boldsymbol{\theta}}_a]_{N_s+1}^2 - \mathbf{x}_{t,1}^T \mathbf{x}_{t,1} + \boldsymbol{\alpha}^T \boldsymbol{\alpha} + 2(\mathbf{x}_{t,1} - \boldsymbol{\alpha})^T [\hat{\boldsymbol{\theta}}_a]_{1:N_s} \\ ([\hat{\boldsymbol{\theta}}_a]_{1:N_s} - \boldsymbol{\alpha}) \odot ([\hat{\boldsymbol{\theta}}_a]_{N_s+2:2N_s+1} - \dot{\boldsymbol{\alpha}}) \\ \begin{pmatrix} [\hat{\boldsymbol{\theta}}_a]_{N_s+1} [\hat{\boldsymbol{\theta}}_a]_{2N_s+2} - \dot{\mathbf{x}}_{t,1}^T \mathbf{x}_{t,1} + \dot{\boldsymbol{\alpha}}^T \boldsymbol{\alpha} \\ +(\dot{\mathbf{x}}_{t,1} - \boldsymbol{\alpha})^T [\hat{\boldsymbol{\theta}}_a]_{1:N_s} + (\mathbf{x}_{t,1} - \boldsymbol{\alpha})^T [\hat{\boldsymbol{\theta}}_a]_{N_s+2:2N_s+1} \end{pmatrix} \end{bmatrix} \tag{17}$$

Then, the final estimate of $\boldsymbol{\theta} = [\mathbf{x}^T, \dot{\mathbf{x}}^T]^T$

$$\hat{\boldsymbol{\theta}} = \begin{bmatrix} diag\{sgn([\hat{\boldsymbol{\theta}}_a]_{1:N_s} - \boldsymbol{\alpha})\} \sqrt{[\hat{\boldsymbol{\theta}}_b]_{1:N_s}} + \boldsymbol{\alpha} \\ [\hat{\boldsymbol{\theta}}_b]_{N_s+1:2N_s} \oslash (\hat{\mathbf{x}} - \boldsymbol{\alpha}) + \dot{\boldsymbol{\alpha}} \end{bmatrix} \tag{18}$$

Where sgn (*) denotes the sign function and ⊘ denotes element-wise division. The details of the derivation and expression of (12) and (16) can be referred to Yang et al. [10].

(25)

For the convenience of the description, we will abbreviate the improved 2SWLS method with distance-dependent noises as Ddn-2SWLS in this paper.

Ddn-2SWLS has strong robustness and relatively accurate results as the Aux-2SWLS method under low noise levels, but it will deviate from CRLB quickly like the measurement noise increases. This is due in large part to the fact that with the increase of noise, the reasonable approximation of ignoring high-order error terms becomes less reasonable [17]. That is, the high-order error will bring significant bias and need to be considered.

3. Bias-Reduced Solution

3.1 Bias Analysis

In order to analyze the high-order error and localization bias, we should compute the error in each stage of the Ddn-2SWLS. According to Yang et al. [10], the error of the stage 1 is

$$\Delta\boldsymbol{\theta}_a = (\mathbf{G}_a^T \mathbf{W}_a^{-1} \mathbf{G}_a)^{-1} \mathbf{G}_a^T \mathbf{W}_a^{-1} (\mathbf{h}_a - \mathbf{G}_a \boldsymbol{\theta}_a) = (\mathbf{G}_a^T \mathbf{W}_a^{-1} \mathbf{G}_a)^{-1} \mathbf{G}_a^T \mathbf{W}_a^{-1} \boldsymbol{\varepsilon}_a \tag{19}$$

Where

$$\mathbf{W}_a = \mathbf{E}(\boldsymbol{\varepsilon}_a \boldsymbol{\varepsilon}_a^T), \boldsymbol{\varepsilon}_a = \mathbf{T}_a \begin{bmatrix} \mathbf{q} \\ \dot{\mathbf{q}} \end{bmatrix} + \begin{bmatrix} \Lambda_a \mathbf{q} \\ \Lambda_a \mathbf{q} \end{bmatrix} \odot \begin{bmatrix} \Lambda_a \mathbf{q} \\ \Lambda_a \dot{\mathbf{q}} \end{bmatrix} \tag{20}$$

Here, E(*) denotes the expectation operator. Ta is related to the $\hat{d}_1^t$ and $\hat{\dot{d}}_1^t$ which are the elements of the initial $\hat{\theta}_a$ and independent of q and $\dot{q}$. In this paper, we are only concerned with the bias brought by the high-order terms of noises and the error caused by the initial estimate of $\hat{\theta}_a$ will be the content to be studied in the future.
For convenience of expression, we set

$$\mathbf{q}^2 = \mathbf{q} \odot \mathbf{q}, \ \dot{\mathbf{q}}^2 = \dot{\mathbf{q}} \odot \dot{\mathbf{q}}, \ \mathbf{q}\dot{\mathbf{q}} = \mathbf{q} \odot \dot{\mathbf{q}} \tag{21}$$

$$(\mathbf{G}_a^T \mathbf{W}_a^{-1} \mathbf{G}_a)^{-1} \mathbf{G}_a^T \mathbf{W}_a^{-1} = \mathbf{A} = \begin{bmatrix} \mathbf{A}_1 & \mathbf{A}_2 \\ \mathbf{A}_3 & \mathbf{A}_4 \end{bmatrix} \tag{22}$$

$$(\mathbf{G}_b^T \mathbf{W}_b^{-1} \mathbf{G}_b)^{-1} \mathbf{G}_b^T \mathbf{W}_b^{-1} = \mathbf{B} = \begin{bmatrix} \mathbf{B}_1 & \mathbf{B}_2 \\ \mathbf{B}_3 & \mathbf{B}_4 \end{bmatrix} \tag{23}$$

where A1, A2, A3 and A4 are matrices of equal size; B1, B2, B3 and B4 are matrices of equal size. $q^2, \dot{q}^2$, and $q\dot{q}$ are the second-order terms of noises, which is always ignored by 2SWLS methods for the assumption of small noise level. However, the second order terms of noises actually have a growing significant contribution to the bias with the increase of noises. Consequently, the eff ect of $q^2, \dot{q}^2$, and $q\dot{q}$ will be considered in this paper.
By taking expectations of $\Delta\boldsymbol{\theta}_a$, the bias of $\hat{\boldsymbol{\theta}}_a$ is

$$\mathbf{E}(\Delta\boldsymbol{\theta}_a) = \mathbf{D}_a [\mathbf{E}(\mathbf{q}^2)^T, \mathbf{E}(\dot{\mathbf{q}}^2)^T]^T \tag{24}$$

Where

(25)

$$\mathbf{D}_a = \begin{bmatrix} \mathbf{A}_1 & |\Lambda_a| & \mathbf{0} \\ \mathbf{A}_3 & |\Lambda_a| & \mathbf{0} \end{bmatrix}$$

and $|*|$ denotes the absolute value operator.

Similarly, the error of the stage 2 is

(32)

$$\Delta\boldsymbol{\theta}_b = (\mathbf{G}_b^T\mathbf{W}_b^{-1}\mathbf{G}_b)^{-1}\mathbf{G}_b^T\mathbf{W}_b^{-1}(\mathbf{h}_b - \mathbf{G}_b\boldsymbol{\theta}_b) = \mathbf{B}\boldsymbol{\varepsilon}_b \quad (26)$$

Where

$$\boldsymbol{\varepsilon}_b = \mathbf{T}_b\Delta\boldsymbol{\theta}_a - \begin{bmatrix} [\Delta\boldsymbol{\theta}_a]_{1:N_s+1} \\ [\Delta\boldsymbol{\theta}_a]_{1:N_s+1} \end{bmatrix} \odot \Delta\boldsymbol{\theta}_a \quad (27)$$

Substituting (19) into (26) gives

$$\Delta\boldsymbol{\theta}_b = \mathbf{B}\mathbf{T}_b\mathbf{A}\mathbf{T}_a \begin{bmatrix} \mathbf{q} \\ \dot{\mathbf{q}} \end{bmatrix} + \mathbf{B}\mathbf{T}_b\mathbf{A} \begin{bmatrix} \Lambda_a\mathbf{q} \\ \Lambda_a\mathbf{q} \end{bmatrix} \odot \begin{bmatrix} \Lambda_a\mathbf{q} \\ \Lambda_a\dot{\mathbf{q}} \end{bmatrix}$$
$$-\mathbf{B}\left(\begin{bmatrix} \mathbf{A}_1 & \mathbf{A}_2 \\ \mathbf{A}_1 & \mathbf{A}_2 \end{bmatrix} \mathbf{T}_a \begin{bmatrix} \mathbf{q} \\ \dot{\mathbf{q}} \end{bmatrix}\right) \odot \left(\mathbf{A}\mathbf{T}_a \begin{bmatrix} \mathbf{q} \\ \dot{\mathbf{q}} \end{bmatrix}\right) + \boldsymbol{\Theta}_3\left(\begin{bmatrix} \mathbf{q} \\ \dot{\mathbf{q}} \end{bmatrix}\right) \quad (28)$$

Where $\boldsymbol{\Theta}_3([\mathbf{q}^T, \dot{\mathbf{q}}^T]^T)$ denotes a vector of size $2N_tN_r$ whose each element contains $q_{1,1}^{t_1}\, q_{1,2}^{t_2} \ldots\, q_{N_tN_r}^{t_{N_tN_r}}\, \dot{q}_{1,1}^{t_{N_tN_r+1}}\, \dot{q}_{1,2}^{t_{N_tN_r+2}} \ldots \dot{q}_{N_t,N_r}^{t_{2N_tN_r}}$, where t_1, t_2, ..., $t_{2N_tN_r}$ are natural numbers and $t_1 + t_2 + \cdots + t_{2N_tN_r} \geq 3$. In this paper, $\mathbf{E}(q_{1,1}^{t_1}\, q_{1,2}^{t_2} \ldots\, q_{N_tN_r}^{t_{N_tN_r}}\, \dot{q}_{1,1}^{t_{N_tN_r+1}}\, \dot{q}_{1,2}^{t_{N_tN_r+2}} \ldots \dot{q}_{N_t,N_r}^{t_{2N_tN_r}})=0$ when $t_1 + t_2 + \cdots + t_{2N_tN_r} = 3$, and $\mathbf{E}(q_{1,1}^{t_1}\, q_{1,2}^{t_2} \ldots\, q_{N_tN_r}^{t_{N_tN_r}}\, \dot{q}_{1,1}^{t_{N_tN_r+1}}\, \dot{q}_{1,2}^{t_{N_tN_r+2}} \ldots \dot{q}_{N_t,N_r}^{t_{2N_tN_r}})$ will be extremely small when $t_1 + t_2 + \cdots + t_{2N_tN_r} \geq 4$. That is to say $\boldsymbol{\Theta}_3([\mathbf{q}^T, \dot{\mathbf{q}}^T]^T)$ can be neglected and $\mathbf{E}(\boldsymbol{\Theta}_3([\mathbf{q}^T, \dot{\mathbf{q}}^T]^T))$ is considered to be **0** in the derivation of bias.

Taking expectations of $\Delta\boldsymbol{\theta}_b$ yields

$$\mathbf{E}(\Delta\boldsymbol{\theta}_b) \approx \mathbf{D}_b[\mathbf{E}(\mathbf{q}^2)^T, \mathbf{E}(\dot{\mathbf{q}}^2)^T\,]^T \quad (29)$$

Where

$$\mathbf{D}_b = \mathbf{B}\mathbf{T}_b\mathbf{D}_a - \mathbf{B}\left(\begin{bmatrix} \mathbf{A}_1 & \mathbf{A}_2 \\ \mathbf{A}_1 & \mathbf{A}_2 \end{bmatrix}\mathbf{T}_a\right) \odot (\mathbf{A}\mathbf{T}_a) \quad (30)$$

Furthermore, we can Taylor expand

(31)

$$\boldsymbol{\theta} = \begin{bmatrix} diag\{sgn([\hat{\boldsymbol{\theta}}_a]_{1:N_s} - \boldsymbol{\alpha})\}\sqrt{[\boldsymbol{\theta}_b]_{1:N_s}} + \boldsymbol{\alpha} \\ [\boldsymbol{\theta}_b]_{N_s+1:2N_s} \oslash (\hat{\mathbf{x}} - \boldsymbol{\alpha}) + \dot{\boldsymbol{\alpha}} \end{bmatrix}$$

around $\hat{\boldsymbol{\theta}}_b$, and the error of $\hat{\boldsymbol{\theta}}$ is found to be

$$\Delta\boldsymbol{\theta} = diag\{sgn([\hat{\boldsymbol{\theta}}_a]_{1:N_s} - \boldsymbol{\alpha}), sgn([\hat{\boldsymbol{\theta}}_a]_{1:N_s} - \boldsymbol{\alpha})\} \quad (32)$$
$$\begin{bmatrix} [\Delta\boldsymbol{\theta}_b]_{1:N_s} \oslash \left(2[\hat{\boldsymbol{\theta}}_b]_{1:N_s}^{\frac{1}{2}}\right) + [\Delta\boldsymbol{\theta}_b]_{1:N_s} \odot [\Delta\boldsymbol{\theta}_b]_{1:N_s} \oslash (8[\hat{\boldsymbol{\theta}}_b]_{1:N_s}^{3/2}) + \mathbf{R}_3([\Delta\boldsymbol{\theta}_b]_{1:N_s}) \\ \begin{pmatrix} [\Delta\boldsymbol{\theta}_b]_{1:N_s} \odot [\hat{\boldsymbol{\theta}}_b]_{N_s+1:2N_s} \oslash (-2[\hat{\boldsymbol{\theta}}_b]_{1:N_s}^{3/2}) + [\Delta\boldsymbol{\theta}_b]_{N_s+1:2N_s} \oslash [\hat{\boldsymbol{\theta}}_b]_{1:N_s}^{1/2} \\ +[\Delta\boldsymbol{\theta}_b]_{1:N_s} \odot [\Delta\boldsymbol{\theta}_b]_{1:N_s} \odot (-3[\hat{\boldsymbol{\theta}}_b]_{N_s+1:2N_s}) \oslash (8[\hat{\boldsymbol{\theta}}_b]_{1:N_s}^{\frac{5}{2}}) \\ +[\Delta\boldsymbol{\theta}_b]_{1:N_s} \odot [\Delta\boldsymbol{\theta}_b]_{N_s+1:2N_s} \oslash (2[\hat{\boldsymbol{\theta}}_b]_{1:N_s}^{3/2}) + \mathbf{R}'_3(\Delta\boldsymbol{\theta}_b) \end{pmatrix} \end{bmatrix}$$

where $\mathbf{R}_3([\Delta\boldsymbol{\theta}_b]_{1:N_s})$ and $\mathbf{R}'_3(\Delta\boldsymbol{\theta}_b)$ are Lagrange remainder terms of $[\boldsymbol{\theta}]_{1:N_s}$ and $[\boldsymbol{\theta}]_{N_s+1:2N_s}$ respectively. Since $\mathbf{R}_3([\Delta\boldsymbol{\theta}_b]_{1:N_s})$ and $\mathbf{R}'_3(\Delta\boldsymbol{\theta}_b)$ are two specific forms of $\boldsymbol{\Theta}_3([\mathbf{q}^T, \dot{\mathbf{q}}^T]^T)$ they are neglected and their mathematical expectations are considered to be **0**.

The bias will then be

$$\mathrm{E}(\Delta\theta) \approx \mathrm{C}\begin{bmatrix} E([\Delta\boldsymbol{\theta}_b]_{1:N_s}) \\ E([\Delta\boldsymbol{\theta}_b]_{N_s+1:2N_s}) \\ E([\Delta\boldsymbol{\theta}_b]_{1:N_s} \odot [\Delta\boldsymbol{\theta}_b]_{1:N_s}) \\ E([\Delta\boldsymbol{\theta}_b]_{1:N_s} \odot [\Delta\boldsymbol{\theta}_b]_{N_s+1:2N_s}) \end{bmatrix} = \mathbf{D}_c \begin{bmatrix} \mathbf{E}(\mathbf{q}^2) \\ \mathbf{E}(\dot{\mathbf{q}}^2) \end{bmatrix} \quad (33)$$

Where

$$\mathbf{D}_c \approx \mathbf{C}\begin{bmatrix} \mathbf{D}_b \\ [\mathbf{B}_1, \mathbf{B}_2]\mathbf{T}_b\mathbf{A}\mathbf{T}_a \odot [\mathbf{B}_1, \mathbf{B}_2]\mathbf{T}_b\mathbf{A}\mathbf{T}_a \\ [\mathbf{B}_1, \mathbf{B}_2]\mathbf{T}_b\mathbf{A}\mathbf{T}_a \odot [\mathbf{B}_3, \mathbf{B}_4]\mathbf{T}_b\mathbf{A}\mathbf{T}_a \end{bmatrix} \quad (34)$$

$$\mathbf{C} = \begin{bmatrix} diag\{\boldsymbol{\beta} \oslash (2[\hat{\boldsymbol{\theta}}_b]_{1:N_s}^{1/2})\} & diag\{\boldsymbol{\beta} \odot [\hat{\boldsymbol{\theta}}_b]_{N_s+1:2N_s} \oslash (-2[\hat{\boldsymbol{\theta}}_b]_{1:N_s}^{3/2})\} \\ \mathbf{0} & diag\{\boldsymbol{\beta} \oslash [\hat{\boldsymbol{\theta}}_b]_{1:N_s}^{1/2}\} \\ diag\{\boldsymbol{\beta} \oslash (8[\hat{\boldsymbol{\theta}}_b]_{1:N_s}^{3/2})\} & diag\left\{\boldsymbol{\beta} \odot (-3[\hat{\boldsymbol{\theta}}_b]_{N_s+1:2N_s}) \oslash (8[\hat{\boldsymbol{\theta}}_b]_{1:N_s}^{5/2})\right\} \\ \mathbf{0} & diag\{\boldsymbol{\beta} \oslash (2[\hat{\boldsymbol{\theta}}_b]_{1:N_s}^{3/2})\} \end{bmatrix}^T$$

$$\boldsymbol{\beta} = \mathrm{sgn}([\hat{\boldsymbol{\theta}}_a]_{1:N_s} - \boldsymbol{\alpha})$$

We do not know the exact value of $[\mathrm{q}^\mathrm{T}, \dot{\mathrm{q}}^\mathrm{T}]^\mathrm{T}$ in practice, so we cannot calculate $\Delta\theta$ directly. However, the bias $\mathrm{E}(\Delta\theta)$ consisting of the second-order term of $[\mathrm{q}^\mathrm{T}, \dot{\mathrm{q}}^\mathrm{T}]^\mathrm{T}$ is linearly related to $[\mathrm{E}(\mathrm{q}^2)^\mathrm{T}, \mathrm{E}(\dot{\mathrm{q}}^2)^\mathrm{T}]^\mathrm{T}$, which is actually a vector consisting of the main diagonal elements of Q and can be estimated by $\hat{\theta}$. In addition, the first-order error term $\Delta\theta_1 = [\Delta\mathrm{x}_1^\mathrm{T}, \Delta\dot{\mathrm{x}}_1^\mathrm{T}]^\mathrm{T}$ in $\Delta\theta$ can be introduced in 2SWLS and derived by solving the new linear equations without the second-order and higher-order error terms. In this way, we can derive the first-order error term $\Delta\theta_1$ and the second-order error term $\Delta\theta_2 = E(\Delta\theta)$ and subtract them from $\hat{\theta}$ to improve accuracy and reduce the bias.

3-2- Bias Reduction

On the basis of the former analysis, we firstly derive the first-order error term $\Delta\theta_1$.

we set $x_0 = x - \Delta x_1$ and $\dot{x}_0 = \dot{x} - \Delta\dot{x}_1$. Then the distance between the jth receiver and target can be approximated by the Taylor expansion without the second-order and higher-order error terms as

$$d_j^r = \|\mathbf{x} - \Delta\mathbf{x}_1 - \mathbf{x}_{r,j}\| \approx d_j^{(0)} + \mathbf{d}_j^{(1)T}\Delta\mathbf{x}_1 \quad (35)$$

where

$$d_j^{(0)} = \|\mathbf{x} - \mathbf{x}_{r,j}\| \quad (36)$$

$$\mathbf{d}_j^{(1)} = -(\mathbf{x} - \mathbf{x}_{r,j})/\|\mathbf{x} - \mathbf{x}_{r,j}\|$$

Moreover, substituting $\mathbf{x}_0 = \mathbf{x} - \Delta\mathbf{x}_1$ and $d_{i,j} = r_{i,j} - q_{i,j}$ into $\|\mathbf{x}_0 - \mathbf{x}_{t,i}\|^2 = (d_{i,j} - d_j^r)^2$ and ignoring the second-order error term give

$$\mathbf{x}_{r,j}^T\mathbf{x}_{r,j} - \mathbf{x}_{t,i}^T\mathbf{x}_{t,i} + r_{i,j}^2 + 2(\mathbf{x}_{t,i} - \mathbf{x}_{r,j})^T\mathbf{x} - 2r_{i,j}d_j^{(0)} \quad (37)$$

$$-2(r_{i,j}\mathbf{d}_j^{(1)} + \mathbf{x}_{t,i}^T - \mathbf{x}_{r,j}^T)\Delta\mathbf{x}_1 \approx 2(r_{i,j} - d_j^{(0)})q_{i,j}$$

(37) can be expressed in matrix form,

$$\boldsymbol{\varepsilon}_{c,1} = \mathbf{h}_{c,1} - \mathbf{G}_{c,1}\Delta\boldsymbol{\theta}_1 \quad (38)$$

Where

$$\mathbf{G}_{c,1} = [2\mathbf{G}_1, \mathbf{0}_{N_r\times N_r,N_s}] \quad (39)$$

$$[\mathbf{G}_1]_{n,1:N_s} = \left(r_{i,j}\mathbf{d}_j^{(1)} + \mathbf{x}_{t,i} - \mathbf{x}_{r,j}\right)^T$$

$$[\mathbf{h}_{c,1}]_n = \mathbf{x}_{r,j}^T\mathbf{x}_{r,j} - \mathbf{x}_{t,i}^T\mathbf{x}_{t,i} + r_{i,j}^2 + 2(\mathbf{x}_{t,i} - \mathbf{x}_{r,j})^T\mathbf{x} - 2r_{i,j}d_j^{(0)}$$

$$n = (i-1)N_r + j$$

$$\boldsymbol{\varepsilon}_{c,1} \approx \mathbf{T}_{c,1}[\mathbf{q}^T, \dot{\mathbf{q}}^T]^T$$

$$\mathbf{T}_{c,1} = [2\mathbf{T}_1, \mathbf{0}_{N_r\times N_r,N_r\times N_r}]$$

$$\mathbf{T}_1 = diag\left\{r_{1,1} - d_1^{(0)}, \dots, r_{1,N_r} - d_{N_r}^{(0)}, \dots, r_{N_tN_r} - d_{N_r}^{(0)}\right\}$$

Differentiating both sides of (37) with respect to time, we obtain

$$\dot{\mathbf{x}}_{r,j}^T\mathbf{x}_{r,j} - \dot{\mathbf{x}}_{t,i}^T\mathbf{x}_{t,i} + \dot{r}_{i,j}r_{i,j} + (\dot{\mathbf{x}}_{t,i} - \dot{\mathbf{x}}_{r,j})^T\mathbf{x} + (\mathbf{x}_{t,i} - \mathbf{x}_{r,j})^T\dot{\mathbf{x}} - \dot{r}_{i,j}d_j^{(0)} - r_{i,j}\dot{d}_j^{(0)} \quad (40)$$

$$-2(\dot{r}_{i,j}\mathbf{d}_j^{(1)} + r_{i,j}\dot{\mathbf{d}}_j^{(1)} + \dot{\mathbf{x}}_{t,i} - \dot{\mathbf{x}}_{r,j})^T\Delta\mathbf{x}_1 - 2(r_{i,j}\mathbf{d}_j^{(1)} + \mathbf{x}_{t,i} - \mathbf{x}_{r,j})^T\Delta\dot{\mathbf{x}}_1$$

$$\approx 2(\dot{r}_{i,j} - \dot{d}_j^{(0)})q_{i,j} + 2(r_{i,j} - d_j^{(0)})\dot{q}_{i,j}$$

where

$$\dot{d}_j^{(0)} = (\dot{\mathbf{x}} - \dot{\mathbf{x}}_{r,j})^T(\mathbf{x} - \mathbf{x}_{r,j})/d_j^{(0)} \quad (41)$$

$$\dot{\mathbf{d}}_j^{(1)} = (\mathbf{x} - \mathbf{x}_{r,j})\dot{d}_j^{(0)}/(d_j^{(0)})^2 - (\dot{\mathbf{x}} - \dot{\mathbf{x}}_{r,j})/d_j^{(0)}$$

Similarly, (40) can be expressed in matrix form,

$$\boldsymbol{\varepsilon}_{c,2} = \mathbf{h}_{c,2} - \mathbf{G}_{c,2}\Delta\boldsymbol{\theta}_1 \quad (42)$$

where

$$\mathbf{G}_{c,2} = [\dot{\mathbf{G}}_1, \mathbf{G}_1] \quad (43)$$

$$[\dot{\mathbf{G}}_1]_{n,1:N_s} = (\dot{r}_{i,j}\mathbf{d}_j^{(1)} + r_{i,j}\dot{\mathbf{d}}_j^{(1)} + \dot{\mathbf{x}}_{t,i} - \dot{\mathbf{x}}_{r,j})^T$$

$$[\mathbf{h}_{c,2}]_n = \dot{\mathbf{x}}_{r,j}^T\mathbf{x}_{r,j} - \dot{\mathbf{x}}_{t,i}^T\mathbf{x}_{t,i} + \dot{r}_{i,j}r_{i,j} + (\dot{\mathbf{x}}_{t,i} - \dot{\mathbf{x}}_{r,j})^T\mathbf{x}$$

$$+(\mathbf{x}_{t,i} - \mathbf{x}_{r,j})^T\dot{\mathbf{x}} - \dot{r}_{i,j}d_j^{(0)} - r_{i,j}\dot{d}_j^{(0)}$$

$$\boldsymbol{\varepsilon}_{c,2} \approx \mathbf{T}_{c,2}[\mathbf{q}^T, \dot{\mathbf{q}}^T]^T$$

$$\mathbf{T}_{c,2} = [\dot{\mathbf{T}}_1, \mathbf{T}_1]$$

$$\dot{\mathbf{T}}_1 = diag\left\{\dot{r}_{1,1} - \dot{d}_1^{(0)}, \dots, \dot{r}_{1,N_r} - \dot{d}_{N_r}^{(0)}, \dots, \dot{r}_{N_t,N_r} - \dot{d}_{N_r}^{(0)}\right\}$$

In (37) and (40), $\mathbf{x}$ and $\dot{\mathbf{x}}$ are replaced by the result $\hat{\boldsymbol{\theta}}$ in (18). From (38) and (42), the integrated equation can be expressed in the matrix form as

$$\boldsymbol{\varepsilon}_c = \mathbf{h}_c - \mathbf{G}_c\Delta\boldsymbol{\theta}_1 \quad (44)$$

Where

$$\mathbf{G}_c = [\mathbf{G}_{c,1}^T, \mathbf{G}_{c,2}^T]^T \quad (45)$$

$$\mathbf{h}_c = [\mathbf{h}_{c,1}^T, \mathbf{h}_{c,2}^T]^T$$

$$\boldsymbol{\varepsilon}_c \approx \mathbf{T}_c[\mathbf{q}^T, \dot{\mathbf{q}}^T]^T$$

$$\mathbf{T}_c = [\mathbf{T}_{c,1}^T, \mathbf{T}_{c,2}^T]^T$$

The solution of (44) is calculated using WLS as

$$= (\mathbf{G}_c^T \mathbf{W}_c^{-1} \mathbf{G}_c)^{-1} \mathbf{G}_c^T \mathbf{W}_c^{-1} \mathbf{h}_c \quad (46)$$

Where $\mathbf{W}_c \approx \mathbf{T}_c Q \mathbf{T}_c^T$.

The derivation process of $\Delta\hat{\theta}_1$ ignores the second-order error terms, so the derived $\Delta\hat{\theta}_1$ is the first-order error term of Ddn-2SWLS, which needs to be eliminated from the original estimate $\hat{\theta}$.

Then, we recalculate Ta, Wa and Wb by (13) and (17) on the basis of $\hat{\theta}$, and get the second-order error term $\Delta\theta_2$ from (24), (29) and (33). Finally, the bias-reduced solution of $\hat{\theta}_0 = [\hat{x}_0^T, \hat{\dot{x}}_0^T]^T$ can be given by

$$\hat{\boldsymbol{\theta}}_0 = \hat{\boldsymbol{\theta}} - \Delta\hat{\boldsymbol{\theta}}_1 - \Delta\boldsymbol{\theta}_2 \quad (47)$$

This paper is concerned with the root mean square position error (RMSPE), the root mean square velocity error (RMSVE), the position bias (Pbias) and the velocity bias (Vbias) to evaluate the performance of target localization. RMSPE, RMSVE, Pbias and Vbias are defined as

$$RMSPE = \sqrt{E(\|\mathbf{x}_0 - \hat{\mathbf{x}}_0\|^2)} \quad (48)$$

$$RMSVE = \sqrt{E(\|\dot{\mathbf{x}}_0 - \hat{\dot{\mathbf{x}}}_0\|^2)}$$

$$Pbias = \|\mathbf{E}(\mathbf{x}_0 - \hat{\mathbf{x}}_0)\|$$

$$Vbias = \|\mathbf{E}(\dot{\mathbf{x}}_0 - \hat{\dot{\mathbf{x}}}_0)\|$$

respectively. From the former part of bias analysis, we can see that the elimination of the second-order error term $\mathbf{E}(\Delta\boldsymbol{\theta})$ can lower the Pbias and Vbias. At the same time, At the same time, the bias-reduced solution eliminating the first-order and second-order error terms in (48) can theoretically improve the performance of RMSPE and RMSVE very well.

4. Numerical Simulations

The simulations are performed to compare the relative localization accuracy and bias value of different methods. To evaluate the comprehensive performance of different methods, we simulate the near-field localization and the far-field localization, respectively. The near-field case means the target position is roughly in the spatial structure formed by the illuminators and receivers, while the far-field case refers to the target located outside the spatial structure formed by the illuminators and receivers.

For near-field case in this paper, we set the target randomly within a sphere, of which center is the coordinate origin and radius is R/2. The illuminators and receivers are placed randomly between the two spherical surfaces whose centers are the origin and radii are R/2 and R, respectively, as shown in Figure 2, where cubes, tetrahedron and, sphere represent illuminators, receivers and target respectively. Meanwhile, the velocities of the target, the illuminators and, the receivers are random vectors whose lengths are not greater than V. Here, R and V are two constants to represent distance and velocity respectively.

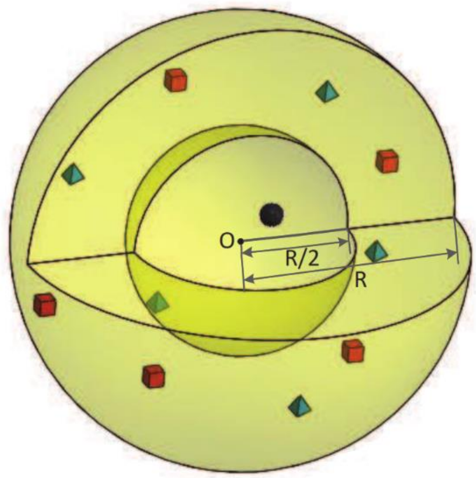


Fig. 2 An example of spatial structure in near-field case.

For the far-field case in this paper, we set the target randomly between the two spherical surfaces with radius R/2 and R respectively, and the illuminators and receivers randomly within the sphere with radius R/2, as shown in Figure 3. The velocities are random vectors no greater than V.

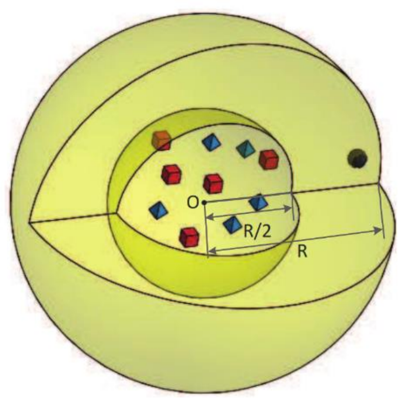
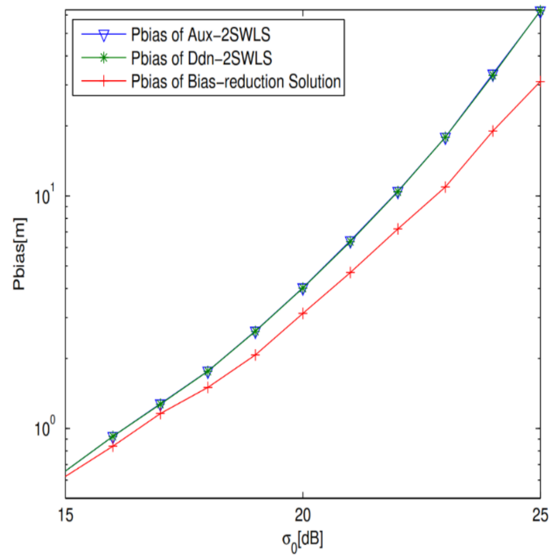


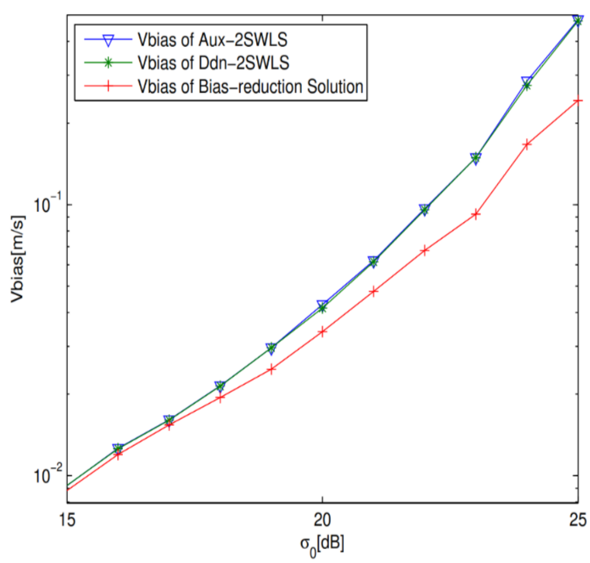
Fig. 3 An example of spatial structure in far-field case.

Considering an illuminator of opportunity passive radar consisting of five illuminators and five receivers in 3-D space, we set R=2000m, V=20m/s and, η = 0.01. The simulation results are obtained via 10000 Monte Carlo runs. The simulation results of RMSPE, RMSVE, Pbias and Vbias of

Aux-2SWLS, Ddn-2SWLS and the proposed bias-reduction solution concerning σ_0 in different cases are given in Figure 4, 5, 6 and 7.

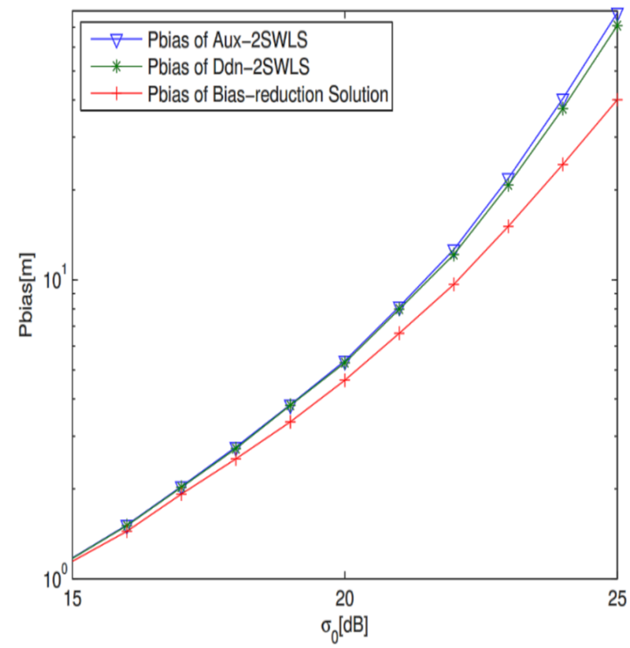


(a) Comparison of the Pbias.

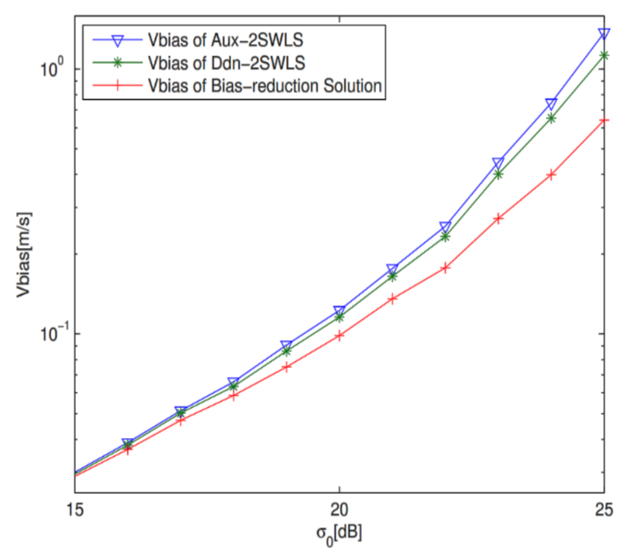


(b) Comparison of the Vbias

Fig. 4 Comparison of the bias of Aux-2SWLS, Ddn-2SWLS and the proposed bias-reduction solution with regard to σ_0 in the near-field case: (a) Pbias and (b) Vbias.



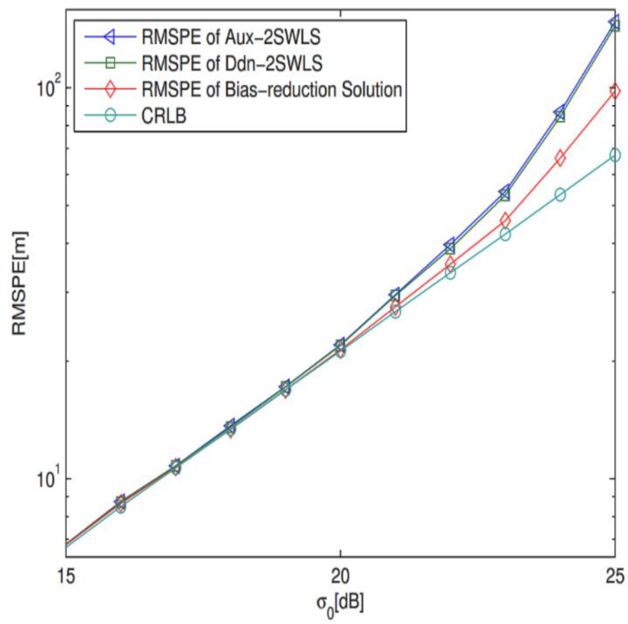
(a) Comparison of the Pbias.



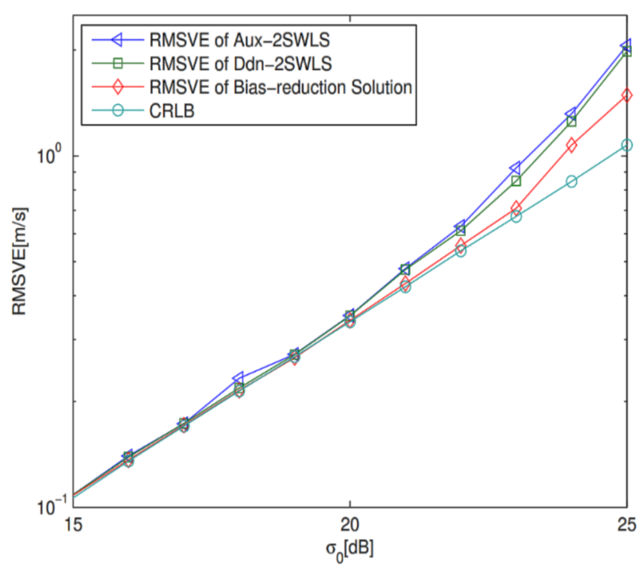
(b) Comparison of the Vbias.

Fig. 5 Comparison of the bias of Aux-2SWLS, Ddn-2SWLS and the proposed bias-reduction solution with regard to σ_0 in the far-field case: (a) Pbias and (b) Vbias.

Figure 4 and Figure 5 illustrate the bias of the Aux-2SWLS method, the Dnd-2SWLS method and the proposed bias-reduction method at different noise levels. As we can see from Figure 4 and Figure 5, both in near-field and far-field cases, Aux-2SWLS and Ddn-2SWLS have a similar bias, while the bias-reduction solution could reduce the bias to a considerable extent compared to Aux-2SWLS and Ddn-2SWLS, especially at higher noise levels. For instance, when σ_0 is 25dB, the Pbias and Vbias of the bias-reduction solution are about half of these of Aux-2SWLS and Ddn-2SWLS. This proves the effectiveness of the proposed bias-reduction solution.



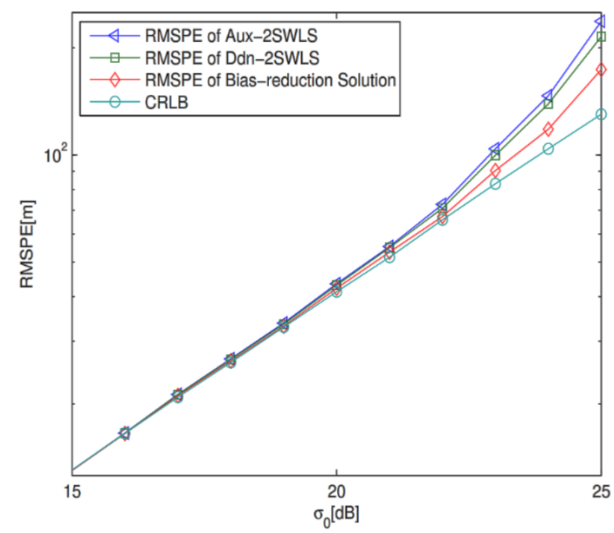
(a) Comparison of the RMSPE.



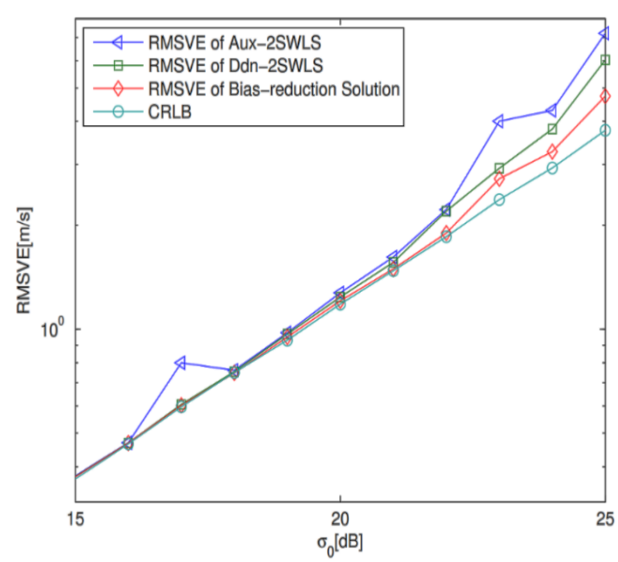
(b) Comparison of the RMSVE.

Fig. 6 Comparison of the RMSPE and RMSVE of Aux-2SWLS, Ddn-2SWLS and the proposed bias-reduction solution with regard to σ_0 in the near-field case: (a) RMSPE and (b) RMSVE.

We can see from Figure 6 and Figure 7 that there is an apparent difference between CRLB curves and simulation results of different methods since σ_0 is above 20dB. When _0 is larger than 20dB, not at all times but in general, the RMSPE and RMSVE curves of Ddn-2SWLS are obviously closer to the CRLB than Aux-2SWLS for the consideration of distance-dependent noises, especially for the far-field case. The locations of illuminators, receivers, and targets in this simulation are limited in different spheres while, theoretically, the difference between the curves of Aux-2SWLS and Ddn-2SWLS will be bigger when the spatial geometry of illuminators, receivers, and target is more complicated. Furthermore, through bias reduction, the localization accuracy has been greatly improved, and it implies the practical significance of the bias-reduction solution at high noise levels.



(a) Comparison of the RMSPE.



(b) Comparison of the RMSVE.

Fig. 7 Comparison of the RMSPE and RMSVE of Aux-2SWLS, Ddn-2SWLS and the proposed bias-reduction solution with regard to σ_0 in the far-field case: (a) RMSPE and (b) RMSVE.

To Find the CRLB for estimating the phase we need the PDF:

$$P(x;\emptyset) = \frac{1}{(2\pi\delta^2)^{\frac{N}{2}}} \exp\left[\frac{-\sum_{n=0}^{N-1}(x[n]-2\pi fn+\emptyset)^2}{2\delta^2}\right] \tag{49}$$

Now taking the log gets rid of the exponential, then taking partial derivative gives:

$$\frac{\partial lnp(x;\emptyset)}{\partial\emptyset} = \frac{-A}{\partial^2}\sum_{n=0}^{N-1}(x[n]\sin(2\pi fn+\emptyset) - \frac{A}{2}\sin(4\pi fn + 2\emptyset))^2 \tag{50}$$

Taking partial derivative again:

$$\frac{\partial^2 lnp(x;\emptyset)}{\partial\emptyset^2} \; \frac{-A}{\partial^2}\sum_{n=0}^{N-1}(x[n]\cos(2\pi fn+\emptyset) - A\cos(4\pi fn + 2\emptyset))^2 \tag{51}$$

Still depends on random vector x so need E{}
Taking the expected value:

$$-E\left\{\frac{\partial^2 lnp(x;\emptyset)}{\partial\emptyset^2}\right\} = E\left\{\frac{-A}{\partial^2}\sum_{n=0}^{N-1}(x[n]\cos(2\pi fn+\emptyset) - A\cos(4\pi fn+2\emptyset))^2\right\} = \frac{-A}{\partial^2}\left(E\{(x[n]\}\cos(2\pi fn+\emptyset) - A\cos(4\pi fn+2\emptyset)\right) \tag{52}$$

$$E\{(x[n]\} = A\cos(2\pi fn+\emptyset)$$

So, plug that in, get a $\cos^2$ term, use trig identity, and get

$$-E\left\{\frac{\partial^2 lnp(x;\emptyset)}{\partial\emptyset^2}\right\} = \frac{A^2}{2\partial^2}\left[\sum_{n=0}^{N-1}1 - \sum_{n=0}^{N-1}\cos(4\pi fn+2\emptyset)\right] \approx \frac{NA^2}{2\partial^2} = N \times SNR \tag{53}$$

f not near to 0 or $\frac{1}{2}$

Now, invert to get CRLB:

$$Var\{\widehat{\emptyset}\} = \frac{1}{N\times SNR} \tag{54}$$

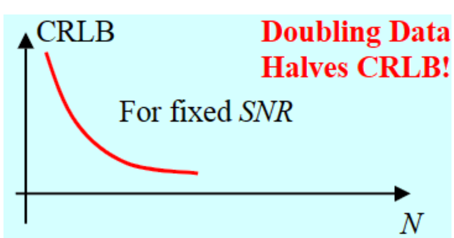


Fig. 8 Doubling data halves CRLB

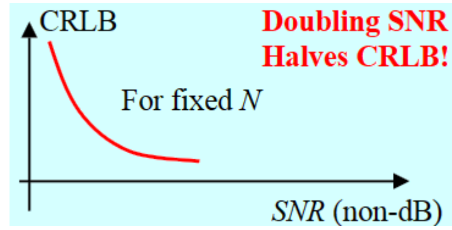


Fig. 9 Halve CRLB for every 3B in SNR

Does an efficient estimator exist for this problem? The CRLB theorem says there is only if

$$\frac{\partial lnp(x;\emptyset)}{\partial\emptyset} = I(\theta)[g(x)-\theta] \tag{55}$$

Our earlier result was:

$$\frac{\partial lnp(x;\emptyset)}{\partial\emptyset} = \frac{-A}{\partial^2}\sum_{n=0}^{N-1}(x[n]\sin(2\pi fn+\emptyset) - \frac{A}{2}\sin(4\pi fn + 2\emptyset))^2 \tag{56}$$

We will see later though, an estimator for which $Var\{\widehat{\Phi}\}$→CRLB as N→∞ or as SNR→∞

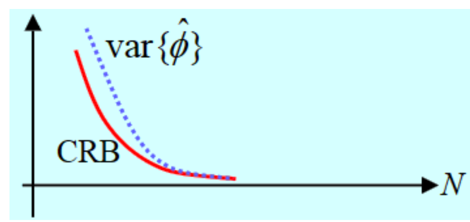


Fig. 10 $Var\{\widehat{\Phi}\}$→CRLB as N→∞ or as SNR→∞

Such an estimator is called an asymptotically efficient estimator

5. Conclusions

Considering the realistic distance-dependent noises and the bias in 2SWLS methods, we developed a bias-reduced solution for target localization in illuminator of opportunity passive radar. This solution is proved by numerical simulations to be effective to reduce the bias and attain the CRLB, especially at higher noise levels. The theoretical performance of the proposed method is derived via second-order error analysis, demonstrating theoretically the effectiveness of the proposed method in reducing the bias and achieving the CRLB under moderate noise. That is to say, the proposed bias-reduced solution is useful for the illuminator of opportunity passive radar localization problem.

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