

# GoF-Based Spectrum Sensing of OFDM Signals over Fading Channels

Seyed Sadra Kashef

Department of Electrical and Computer Engineering, Tarbiat Modares University, Tehran, Iran  
s.kashef@modares.ac.ir

Paeiz Azmi\*

Department of Electrical and Computer Engineering, Tarbiat Modares University, Tehran, Iran  
pazmi@modares.ac.ir

Hamed Sadeghi

Department of Electrical and Computer Engineering, Tarbiat Modares University, Tehran, Iran  
hamed.sadeghi@modares.ac.ir

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## Abstract

Goodness-of-Fit (GoF) based spectrum sensing of orthogonal frequency-division multiplexing (OFDM) signals is investigated in this paper. To this end, some novel local sensing methods based on Shapiro-Wilk (SW), Shapiro-Francia (SF), and Jarque-Bera (JB) tests are first studied. In essence, a new threshold selection technique is proposed for SF and SW tests. Then, three studied methods are applied to spectrum sensing for the first time and their performance are analyzed. Furthermore, the computational complexity of the above methods is computed and compared to each other. Simulation results demonstrate that the SF detector outperforms other existing GoF-based methods over AWGN channels. Furthermore simulation results demonstrate the superiority of the proposed SF method in additive colored Gaussian noise channels and over fading channel in comparison with the conventional energy detector.

**Keywords:** Cognitive Radio, Spectrum Sensing, Goodness-of-Fit (GoF), Orthogonal Frequency Division Multiplexing (OFDM).

## 1. Introduction

The motivation for presentation of Cognitive Radio (CR) is the increasing need for higher bandwidth in wireless communications despite limited or licensed spectrum resources. Licensed spectrum is allocated over long time periods and is intended to be used only by licensed users. Different measurements of spectrum utilization have shown significant unused resources in three dimensions of frequency, time, and space [1]. Discovering these underutilized spectrum sources is the main idea behind CR by reusing spectrum holes in an opportunistic way [2]. In a CR network, spectrum sensing (SS) is the main duty of each CR user to find the unused spectrum, or equivalently, the primary users (PUs). One of the most challenging problems in this area is to find a solution to detect the existence and absence of PUs in the wireless communication [3].

Reliable PU detection problem is the main end of many SS algorithm proposals. For example, in the presence of PUs, when PU's signal is known, the best sensing method is matched filtering. However, when the primary signal is not perfectly known, energy detection (ED) method can be used instead of matched filtering [4]. In situations that SNR is low, distinguishing between PUs and noise is not simple. ED method is often considered for SS because of simplicity and admirable performance over SNR situations. However, uncertainty of the noise

power quickly destroys the performance of ED. In practice, noise is an summation of various sources which can be changed significantly; therefore, usually uncertainty of noise variance exists and ranges about 1 to 2 dB [5]. By knowing characteristics of the incoming signals, different algorithms were recommended to increase the performance of ED consisting of waveform-based sensing and cyclostationarity-based sensing (See [6]). According to mathematical statistics, these methods are part of parametric hypothesis testing. It means that incorrect assumption about the received signals' parameters will degrade the performance. Accordingly, it is not easy to perform detecting the signal without key information. The appropriate signal features should be reported in the feature detecting methods (e.g. cyclostationarity). On the other hand, having information about the PUs is impossible practically in a CR receiver [7]. For instance, the matched filtering method, which provides the maximum SNR at the output of the detector, requires the exact knowledge of PU waveform. In addition, in the cyclostationary feature detection method, the cycle frequency of the primary signal should be known completely. Some PU detection algorithms based on statistical properties of eigenvalues of the covariance matrix of the received signals were devised in [8] in an attempt to compensate for the weaknesses of the above methods. However, the order of computational

\* Corresponding Author

complexity of these algorithms is generally huge, which limits their practicality in CR devices [9].

To compensate weakness of the above methods, some PU detection methods have been proposed in the literatures. Recently, a higher-order-statistics (HOS) techniques was applied in [9] in SS problem for a reliable detection of PUs in the low SNR situations. In addition, a powerful sensing algorithm based on JB test [10] has been devised in [9], which is inherently a GoF testing problem. It has been shown that this method provides a high detection performance in very low SNRs [9]. Several GoF-based sensing methods have been recently proposed in the literatures [11],[12],[13],[14], where they provide superior performance in challenging opportunistic applications. Note that GoF-based sensing methods do not require any prior knowledge about the transmitted PU signals.

In this paper, we review three GoF techniques: Shapiro-Wilk (SW), Shapiro-Francia (SF), and Jarque-Bera (JB) tests. Then, we propose two SS methods based on SW and SF methods. Since these tests are kinds of Gaussianity tests, we assume that the distribution of channel noise is Gaussian. We compare these algorithms with each other, and also with a conventional GoF-based sensing approach, i.e., the Anderson-Darling (AD) test. In essence, we show that the computational complexities of the proposed methods are lower than the AD method. Also, we show that SF is faster than JB and SW since its computational complexity is lower. Furthermore, we will show through simulation results that SF outperforms the other candidates in different SNR values, signal sample sizes, and channel characteristics. Thus, the SF detector can effectively contribute to future CR networks.

It is straightforward to show that the first-order distribution of OFDM signals will converge to a Gaussian variable [15]. Suppose that an OFDM-based primary user signal is already present in the spectrum. In this case, the Gaussianity-based SS techniques would fail in PU detection; they will wrongly decide the hypothesis  $H_0$  instead of  $H_1$ . Thus, we propose using FFT block as a preprocessing method to fix this problem when using GoF-based Gaussianity tests for sensing of OFDM signals. The idea is the fact that OFDM signals do not show Gaussian behavior in the frequency domain [16].

The organization of this paper is as follows. Section II introduces the GoF-based sensing and colored noise concept. Section III presents the statistical GoF tests. In Section IV, we propose the GoF-based spectrum sensing method. Section 5 presents the simulation results and finally, Section 6 concludes the paper.

## 2. GoF-based Spectrum Sensing Methods

Spectrum sensing algorithms must detect the presence/absence of PUs as quick as possible. If CR decides that the considered channel is empty, then it uses that frequency band for opportunistic communication. However, if CR misses the PU detection, it will cause a

harmful interference to PU. Thus, the detection performance of the SS algorithm is an important factor in CR networks.

It is well-known that SS is a binary hypothesis testing problem as follows,

$H_0$ : Presence of noise only

$H_1$ : Presence of Primary User + noise

Let  $\mathbf{y} = \{y_i\}_{i=1}^N$  denotes  $N$  local time-domain observation samples collected at each CR. Without loss of generality,  $y_i$ ,  $i = 1, \dots, N$ , are assumed to be real-valued. In situations that there is no primary transmission,  $y_1, \dots, y_N$  are only the noise samples. In this situation, they could be considered as an independent and identically distributed (i.i.d.) sequence with cumulative distribution function  $F_0(\mathbf{y})$ . However, based on the kind of modulation and communication link characteristics, the gathered samples  $y_1, \dots, y_N$  may not have distribution  $F_0(\mathbf{y})$  when the PU's signal is present. In other words, this situation occurs when the received samples are not coming from the distribution function  $F_0(\mathbf{y})$ . Thus, the null hypothesis can be described as:

$H_0$  :  $\mathbf{y}$  is an i.i.d. sequence obtained from distribution  $F_0(\mathbf{y})$ .

Furthermore, the alternative hypothesis ( $H_1$ ) is the situation in which the received samples  $\mathbf{y}$  do not form an i.i.d. sequence coming from distribution  $F_0(\mathbf{y})$ . There is no need for the knowledge of any information about the PU's signal in the above-mentioned GoF-based hypothesis testing problem and the type of noise distribution is the only assumption for detection.

Due to the presence of a colored channel interferer or some other reasons, the conventional white Gaussian noise may become colored. Therefore, in presence of colored noise the performance of SS methods is degraded. Independence of received samples is negligible in GoF based SS problem. The model of colored noise will be introduced in the next section.

### 2.1 Colored noise

It is clear that knowing the exact covariance matrix of the noise is necessary for the conventional energy detection method in colored noise, which is impractical in AWGN channel. The uncertainty of covariance matrix will lead to performance degradation because of inaccurate estimate of the noise parameters. According to the work in [17], here some new definitions on noise uncertainty in colored noise is introduced.

Based on [18], the colored Gaussian noise can be seen as the output of a single pole recursive filter stimulated by a white Gaussian noise (WGN). In mathematics, it can be expressed as  $w(t) = -\eta w(t-1) + u(t)$ , where  $w(t)$  is the colored noise,  $u(t)$  is the WGN with variance  $\sigma_u^2$  and  $\eta$  ( $|\eta| < 1$ ) is the correlation strength of the noise  $w(t)$ .

To have the exact covariance matrix of the colored noise, the exact  $\sigma_u^2$  and  $\eta$  should be known. Practically, the two parameters should be estimated and thus there are uncertainties. According to what is mentioned in [17] we

assume the estimated parameters  $\hat{\sigma}_u^2, \hat{\eta}$  are the multiples of the actual values  $\sigma_u^2, \eta$ , i.e.  $\hat{\eta} = \beta\eta$  and  $\hat{\sigma}_u^2 = \alpha\sigma_u^2$ . So, we say there are noise uncertainties for signal detection in colored Gaussian noise [19].

## 2.2 Anderson-darling test

The GoF tests quantify a distance between the distribution functions of two sample sets. The transmitted signal assumption is not needed in these tests at all. Anderson-Darling (AD) test is a popular GoF test in statistics that has been applied to SS in [11] and [12]. It will be discussed in the following.

AD test is an extension of the Cramer-von Mises (CM) test, so we will have a short description about the CM statistic.

The CM statistic  $W^2$  is defined by [11],

$$W^2 \triangleq N \int_{-\infty}^{+\infty} (F_Y(y) - F_0(y))^2 dF_0(y). \quad (1)$$

It is obvious there is an important problem in CM statistic which is assigning sufficient weights to the sequences of the distribution  $F_0(y)$ . Anderson and Darling [20] improved the CM statistic by introducing a weighted statistic as follows:

$$A_c^2 \triangleq N \int_{-\infty}^{+\infty} (F_Y(y) - F_0(y))^2 \phi(F_0(y)) dF_0(y), \quad (2)$$

where  $\phi(t)$  is a nonnegative weight function defined over  $0 \leq t \leq 1$ . A common weighting function for AD statistic is

$$\phi(t) = \frac{1}{t(1-t)} \quad (3)$$

Finally, in the AD test, if  $t_0$  is a threshold or critical point to be selected, the null hypothesis  $H_0$  is rejected if and only if  $A_c^2 > t_0$  [11]. Thus, the probability of false alarms under  $H_0$  is:

$$Pr\{A_c^2 \geq t_0 | H_0\} \quad (4)$$

Now, according to this description, it is clear that the two phases of the Anderson-Darling test are as follows:

1. Calculate AD test statistic using equation (2).
2. Determine  $t_0$  (threshold) according to the probability of false alarm or  $\alpha$ .

The calculation of Eq. (2) is not a simple task, so it is not hard to show by breaking the whole integral in (2) into  $n$  parts as follows [11]:

$$A_c^2 = - \frac{\sum_{i=1}^n (2i-1)(\ln z_i + \ln(1-z_{n+1-i}))}{n} - n \quad (5)$$

where:

$$z_i = F_0(y_i) \quad (6)$$

As it is observed in Eq. (6),  $z_i$  or  $F_0(y_i)$  is the cumulative distribution function of noise. It can be shown that the distribution function depends on the variance of the noise. Hence, uncertainty in the noise variance will strongly influence its performance [11].

To overcome this weakness, Blind AD method is proposed which can overcome the weakness of the AD test. In Blind AD method, first of all, one divisor of  $n$  (which is the number of samples) denoted by  $m$  is chosen. Then samples are divided into  $l = \frac{n}{m}$  groups, each containing  $m$  samples. So, we will have:

$$\bar{Y}_j \triangleq \sum_{k=0}^{m-1} \frac{Y_{mj-k}}{m} \quad S_j^2 \triangleq \sum_{k=0}^{m-1} \frac{(Y_{mj-k} - \bar{Y}_j)^2}{m-1} \quad j = 1, \dots, l \quad (7)$$

In these equations,  $\bar{Y}_j$  and  $S_j^2$  are the mean and variance of the samples in  $j$ th group, respectively. To remove the uncertainty effects of noise variances in sensing, a key equation is suggested as follows:

$$X_j \triangleq \frac{\bar{Y}_j}{\frac{S_j}{\sqrt{m}}} \quad j = 1, 2, \dots, l. \quad (8)$$

It can be indicated that the primary user do not send any signal and the received samples only contain noise.  $X_j$  is independent of the noise variance which concludes  $F_{0,m}(y)$ , the cumulative distribution of  $m^{th}$  group, is independent of noise variance as well.

Some works have been done with two-sampled GoF tests for modifying them to SS in [12], [11]. But there is a weakness in two sample tests which is the need for prior samples from channel noise. However, this is the first attempt in this paper to use one-sampled GoF tests in CR networks. Superiority of one-sample tests is that they don't need any prior information or sample about the channel noise. In contrast, in two-sample tests, having at least one noise sample is necessary as a prior sample. This requirement is hard to meet in some busy channels where the assessment of empty spectrum is difficult. Following section will introduce three one-sample tests and then modify them for SS.

## 3. Presentation of Considered GoF Tests

In this section we study three GoF tests, that is, JB, SW and SF. Afterwards, we study their potential for use in SS.

### 3.1 Jarque-bera test

The first considered test is JB. This test is a one-sample GoF technique for measurement of deviation from Gaussianity and is constructed from the sample kurtosis and skewness. The test is entitled JB due to its pioneers Carlos M. Jarque and Anil K. Bera. Its test statistic is given by [10]

$$JB \triangleq \frac{N}{6} \left( S^2 + \frac{(K-3)^2}{4} \right), \quad (9)$$

where  $N$  is the number of samples,  $K$  denotes the sample kurtosis and  $S$  is the skewness of observation samples, defined as:

$$S \triangleq \frac{\hat{\mu}_3}{\hat{\sigma}^3} = \frac{\hat{\mu}_3}{(\hat{\sigma}^2)^{3/2}} = \frac{\frac{1}{N} \sum_{i=1}^N (y_i - \bar{y})^3}{\left( \frac{1}{N} \sum_{i=1}^N (y_i - \bar{y})^2 \right)^{3/2}} \quad (10)$$

$$K \stackrel{\text{def}}{=} \frac{\hat{\mu}_4}{\hat{\sigma}^4} = \frac{\hat{\mu}_4}{(\hat{\sigma}^2)^2} = \frac{\frac{1}{N} \sum_{i=1}^N (y_i - \bar{y})^4}{\left(\frac{1}{N} \sum_{i=1}^N (y_i - \bar{y})^2\right)^2} \quad (11)$$

in which  $\hat{\mu}_3$  and  $\hat{\mu}_4$  are the estimates of the third and fourth central moments, respectively;  $y_i, i = 1, \dots, N$  are the received samples;  $\bar{y}$  is the sample mean and  $\hat{\sigma}^2$  denotes the variance estimation [9].

If we have  $JB > t_0$ , the null hypothesis is rejected. In contrast, the null hypothesis will be accepted if  $JB < t_0$ . The threshold values are computed using the critical values listed in [10].

### 3.2 Shapiro-wilk test

The S-W test relies on the correlation between ‘‘order statistics’’ of observed samples and a Gaussian distribution. The order statistics is used to represent that the data sample has to be classified, in vector form sorted in an increasing order as  $y' = (y_1, \dots, y_N)$ ; where the prime ' denotes the transpose of a vector. The SW test statistic  $W$  is defined as

$$W = \frac{\left(\sum_{i=1}^N a_i y_i\right)^2}{\sum_{i=1}^N (y_i - \bar{y})^2} \quad (12)$$

where  $\bar{y}$  is the sample mean.  $W$  can be defined as a ratio of two estimates of the sample variance, with the estimate in the numerator holding only if the sample is obtained from a Gaussian distribution whereby coefficients  $a_i$  are calculated by linear regression to the expected values of standard Gaussian order statistics. Notably, the expected value of  $W$  converges to zero when the input signal becomes non-Gaussian and the expected value of  $W$  converges to one for Gaussian input signal as the sample size grows [21].

Vector of coefficients  $\mathbf{a}' = (a_1, \dots, a_N)$  is normalized so  $\mathbf{a} \cdot \mathbf{a}' = 1$ , and in the way that they become symmetrically mirrored, that is,  $a_N = -a_1, a_2 = -a_{N-1}$ , and so on. These coefficients exist in known statistical literatures. However, these tables are limited and the main problem is to get all of the coefficients for all  $N$ . Thus, for simplicity, the coefficients are approximated with the following polynomials [21]:

$$a_N = -2.706056u^5 + 4.434685u^4 - 2.071190u^3 - 0.147981u^2 + 0.221157u + c_N \quad (13)$$

$$a_{N-1} = -3.582633u^5 + 5.682633u^4 - 1.752461u^3 - 0.293762u^2 + 0.042981u + c_{N-1} \quad (14)$$

$$a_i = \epsilon^{-1/2} \tilde{m}_i \quad (15)$$

in these equations  $u = N^{-\frac{1}{2}}$ , and Eq. (15) for  $i = 3, \dots, N - 2$  is established.

In (15),  $\tilde{\mathbf{m}}' = (\tilde{m}_1, \dots, \tilde{m}_N)$  denotes a vector of expected values of order statistics of the standard Gaussian random variables which can be approximated by the following expression:

$$\tilde{m}_i = \phi^{-1}\{(i - 3/8)/(N + 1/4)\} \quad (16)$$

in the above equation,  $\phi^{-1}$  is the inverse standard Gaussian distribution function. Also in Eq. (15)

$$\epsilon = \frac{\tilde{\mathbf{m}}' \cdot \tilde{\mathbf{m}} - 2\tilde{m}_N^2 - 2\tilde{m}_{N-1}^2}{1 - 2a_N^2 - 2a_{N-1}^2} \quad (17)$$

Finally, the  $c_i$  values in Eq. (13) and Eq. (14) are determined from vector  $\mathbf{c}' = (c_1, \dots, c_N)$  by

$$\mathbf{c} = \tilde{\mathbf{m}}/(\tilde{\mathbf{m}}' \cdot \tilde{\mathbf{m}})^{1/2} \quad (18)$$

Thus, we can calculate according to the available observations and using the above values in Eq. (12) [21].

### 3.3 Shapiro-francia

Another test based on the correlation of samples is the so-called Shapiro-Francia (SF) [22]. In fact, this test is a modification to the SW test. Assume that the weights for this test are defined as:

$$\mathbf{b}' = \frac{\mathbf{c}'}{(\mathbf{c}' \cdot \mathbf{c})^{1/2}} \quad (19)$$

Then, statistic  $W'$  can be represented as follows,

$$W' = \frac{(\mathbf{b}' \cdot \mathbf{y})^2}{\sum (y_i - \bar{y})^2} = \frac{(\sum b_i y_i)^2}{\sum (y_i - \bar{y})^2} \quad (20)$$

Note that  $\mathbf{c}$  is the vector of expected values of the  $N$  order statistics of the standard Gaussian distribution and  $\mathbf{y}$  is samples' vector [22]. The elements of the vector  $\mathbf{c}$  are defined as equation (16) [22].

## 4. Proposed Spectrum Sensing Methods

Here we apply the presented tests in Section 3 for SS in this section.

In spectrum holes observed samples are obtained independently from the noise distribution. So if we assume that the distribution of channel noise is Gaussian, introduced tests can be used for SS; because, these are Gaussianity tests.

Assume that  $N$  received samples  $\mathbf{y} = \{y_i\}, i = 1, \dots, N$  are generated from a PU in a CR network and the channel is AWGN. When channel is empty and there is no primary signal transmission,  $\mathbf{y} = \mathbf{N}_g$ , where  $\mathbf{N}_g$  is the noise vector. Without loss of generality, we use real part of received samples. Thus, these series are obtained from a real Gaussian distribution with unknown mean and variance. This is the null hypothesis. On the other hand, in transmission of PU signal we have  $\mathbf{y} = \mathbf{H}\mathbf{S} + \mathbf{N}_g$ , where  $\mathbf{S}$  shows the PU signal and  $H$  represents the channel gain between the PU and CR. When PU signal is present, alternative hypothesis is formed. In absence of PU, received signal is Gaussian, because of the AWGN channel assumption. Thus, we gather a Gaussian data sample.

Notice that this method does not need prior information about PU, and the noise uncertainty does not affect its performance.

The SS problem is to accept or reject the alternative hypothesis in favor of the null hypothesis as follows:

$$H_1 : \mathbf{y} \text{ has non-Gaussian distribution} = \text{PU is present}$$

$H_0$  :  $\mathbf{y}$  has Gaussian distribution=PU is absent (noise only)

Therefore, according to our assumption SS changes based on distribution testing. Thus, we can apply the methods of section 3 to SS by testing the distribution of received signal.

#### 4.1 OFDM signal

OFDM based signals have a good performance on various channels without the need to use sophisticated receivers and are ideal for use in broadband wireless channels. This is the reason for using this modulation in many systems such as WLAN.

In OFDM systems we have:

$$x(t) = \sqrt{\frac{P}{N_p}} \sum_k \sum_{n=0}^{N_p-1} c_{n,k} \cdot e^{2i\pi(f_0+n\Delta f)t} \cdot g(t - kT_s) \quad (21)$$

In this equation  $x(t)$  is a signal with multicarrier modulation where  $\{c_{n,k}\}$  is a series of symbols that assume to be i.i.d.  $N_p$ , the number of carriers,  $\Delta f$  the frequency offset between carriers,  $g(t)$  the pulse function and  $P$  is the signal power and  $T_s$  is also a OFDM symbol time. Using the central limit theory [23] and according to the above equation, it can be said that the distribution of OFDM will converge to Gaussian.

When the primary user signal is of a Gaussian type, SS will encounter detection problem, because it will detect and confirm the hypothesis  $H_0$  in the presence of Gaussian OFDM signals and as a result, it will not detect the presence of primary users.

To fix this problem we use FFT operation because OFDM signal in the frequency domain has non-Gaussian properties [24].

After receiving signal, it is transformed to frequency domain. This alters Gaussian properties. It should be noted that the Gaussian noise signal has a Gaussian distribution in frequency domain and it does not lose Gaussianity properties in frequency domain. Proposed GoF tests can be applied to output of FFT block.

#### 4.2 Detection algorithms

We use two different scenarios for evaluating presented tests. In the first scenario for JB, SF and SW tests, the test statistic is calculated and then compared with a threshold. In GoF tests thresholds are determined by using Critical Value (CV) tables. CVs depend on false alarm probability and number of received samples. Table 1 includes CVs for JB test in different situations. If Test Statistic  $> CV$ , the absence of PU is decided. In the presence of PU we have Test Statistic  $< CV$ .

Table 1: Critical values for different  $\alpha$  values and sample size belong to JB test [25].

Sample size (n)	Significance level ( $\alpha$ )		
	0.01	0.05	0.1
100	12.282	5.418	3.680
10	4.821	2.329	1.478

It's should be mentioned that

SS algorithm for JB, SF and SW test can be summarized as follows:

**Step1** Transform received signals into Fourier domain using FFT.

**Step2** Compute the test statistic.

**Step2** reject  $H_0$  if Test Statistic  $< CV$ . In contrast, accepting  $H_0$  and reject  $H_1$  if Test Statistic  $> CV$ .

The proposed sensing diagram is shown in 0.

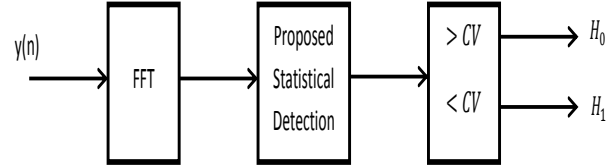


Fig. 1. The proposed spectrum sensing system diagram.

Knowledge of the distribution function of test statistic makes it possible to compare p\_value with  $\alpha$  value directly instead of comparing test statistic with critical values. In statistics, the p-value of a test is defined as the tail integral of the particular instance of the test statistic over the density of the test statistic which is a random variable itself [14]. Assume that a goodness-of-fit test exists with a test statistic  $T$ . Let  $Z_T(\tau)$  be the cumulative distribution of  $T$  under the null hypothesis. p\_value of the test statistic is obtained as follows:

$$p\_value = P(T > \tau | H_0) = 1 - Z_T(\tau) \quad (22)$$

Also, it is obvious

$$P_{fa} = P(T > \tau' | H_0) = 1 - P(T < \tau' | H_0) = 1 - CDF_{(T|H_0)}(\tau') \quad (23)$$

According to Eq. (23) and Eq. (24) we will have

$$p - value \underset{H_0}{\leq} P_{fa} \quad (24)$$

The  $p$ -value acts as an indicator of the confidence of the decision reached by the goodness-of-fit test. A low  $p$ -value ( $p < 0.1$ ) shows a high uncertainty about rejecting the hypothesis while a high  $p$ -value indicates that we are highly confident in rejecting the null hypothesis ( $p < 0.9$ ) [26].

In this case, there is no need to obtain critical values from a significance probability of false alarm and it is enough to directly compare the  $p$ -value with False alarm probability. So, in this situation, we do not need critical value tables.

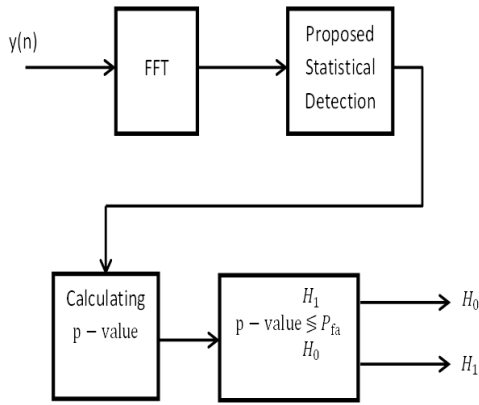


Fig. 2. The proposed spectrum sensing system diagram for  $p$ -value.

In the second scenario, SF and SW tests can use  $p$ -value in detection procedure; because distribution of them is known and derived in [27], [28]. Therefore, another way to implement SF and SW tests for SS is using  $p$ -value. The detection procedure is summarized as follows:

1. Transform the received signals into Fourier domain using FFT.
2. Order the observation samples in an increasing order for calculation of  $W$  or  $W'$  using Eq.(20) or Eq. (12).
3. Compute  $Z_T$  and  $p$ -value using the test statistic distribution.
4. Choose an appropriate significance level  $\alpha$ .
5. Compare  $p$ -value with  $\alpha$ . If  $p$ -value  $< \alpha$ , the null hypothesis will be rejected and  $H_1$  will be confirmed.

The proposed detection diagram changes as Fig.2 for SF and SW tests.

### 4.3 Computational complexity analysis

Here, the computational complexity of the proposed sensing algorithms and AD method as a conventional method are discussed.

Table 2 lists the order of required execution time versus the number of samples for various mathematical operations. Here, to estimate the computational complexity of a test, the highest order of the time complexity is considered.

Table 2: order of computational complexity for different operations [29].

Operations	Complexity Order
Multiplication & Division	$O(N^2)$
Summation & Subtraction	$O(N)$
Square	$O(N^2)$
Natural Logarithm	$O(N^2 \log(N))$
FFT	$O(N \log(N))$

In the AD test, the natural logarithm is used which is more complex than the proposed methods. The proposed methods have just the four main mathematic operators in which multiplication has the maximum complexity. So, the maximum complexity order for the proposed methods

is equal to  $O(N^2)$  because of multiplication complexity. However, the highest order of complexity for AD test is belonging to the natural logarithm which is equal to  $O(N^2 \log(N))$ .

In the proposed methods, an FFT block operation is used for the detection of the OFDM signals. The order of computational complexity for FFT is  $O(N \log(N))$  and is less than complexity order of multiplication. Therefore, there is no change in the maximum complexity order of the proposed methods since the maximum complexity order is still  $O(N^2)$  which is lower than the AD again.

Due to the same complexity order of the proposed algorithms, other methods are also needed to compare their computational complexity. For instance, calculating the number of mathematic operators is another method of measuring the computational complexity of an algorithm in which the operator with the highest complexity is counted. In this method, the maximum complexity order of the proposed methods is related to multiplication (or division).

It is assumed that the number of operators which are not dependent on the number of samples is negligible; since the number of operators increases with the rise of samples numbers.

Table 3 shows the number of multiplication (or division) of the three methods.

Table 3: number of operators for proposed methods.

Proposed method	Operator numbers
JB	$7 \times N$
SW	$5 \times N$
SF	$4 \times N$

In Table 3,  $N$  represents the number of received samples. It is shown that the computational complexity of SF test is less than JB and SW tests. Therefore, SS can be done faster.

We need to do interpolation or extrapolation to find some critical values which do not exist in ready tables. Interpolation or extrapolation increases computational complexity of methods.  $P$ -value solves the problem and decreases the computational complexity of SF and SW methods.

## 5. Simulation Results

In this section, simulation results are demonstrated for different scenarios. Our main goal is to compare the performance of the proposed GoF-based sensing methods with each other and with a conventional approach (i.e. AD).

Before describing the achieved results, it should be noted that WLAN signal is used as the reference signal in this study. The applied WLAN signal in this article is

simulated based on available standards [30]. Obviously, WLAN uses OFDM modulation which is one of the most deployed methods in wireless communication. Taking into account that WLAN signal is based on OFDM, we can generalize our results to other OFDM based signals such as WiMAX and D-VBT.

In implementing the offered methods, an FFT block is used before OFDM detection like OFDM demodulator. It is assumed that  $N$  samples are collected from environment by CRs, written as  $y_i$  for  $i = 1, \dots, N$  which are complex valued. When we calculate proposed tests, without loss of generality, we use real-parts of samples.

Fig. 3 depicts the detection probability of five SS methods including Blind AD, SF, JB, SW and AD. The false alarm probability equals 5% ( $P_{fa} = 5\%$ ) and the environment noise is considered to be Gaussian. The number of available samples from the received signals equals  $N=4000$ , which is equivalent to 50 OFDM symbols. As shown in the Fig. 3, JB and the SF methods have almost similar performance outperforming the other methods.

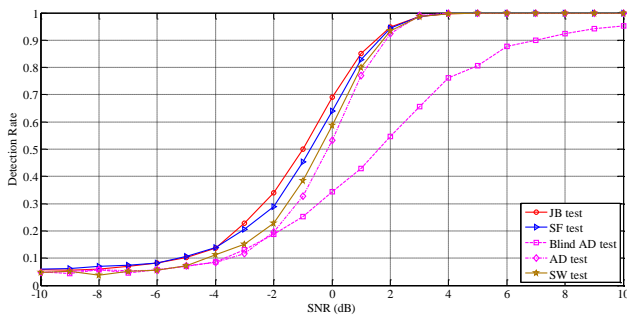


Fig. 3. Detection rate versus SNR value for the simulated WLAN signal ( $N=4000$ ) over AWGN channels ( $P_{fa}=0.05$ ).

The computational complexity of the SF method is much less than AD. Also JB method has high complexity order because of calculating high order statistics. On the other hand, statistic calculations are done only twice for the SF method with less complexity. In addition, JB method needs extrapolation and interpolation. Thus finding CV adds more complexity.

In results of Fig. 4 the same parameters are also taken into account. The only difference is that in this situation, the signal has undergone Rayleigh fading. This figure also supports the fact that the offered SF method has much better performance in comparison with the other methods.

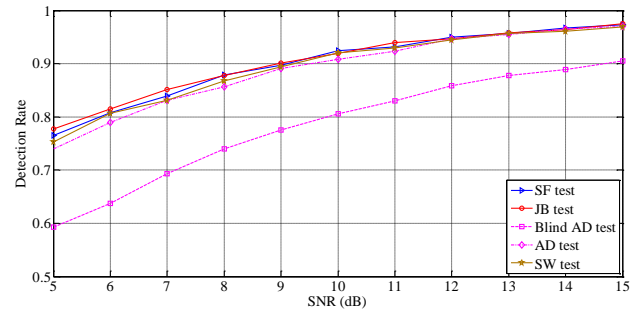


Fig. 4. Detection rate versus SNR for simulated WLAN signal ( $N=4000$ ) over frequency-flat Rayleigh fading channels ( $P_{fa}=0.05$ ).

According to Fig. 5, when the number of received samples reduces to  $N=800$  (i.e. 10 OFDM symbols), the performance of all sensing methods are decreased, while the SF method works better than the others in both low and high SNR values. This observation shows the superiority of the SF sensing method over JB-based method, since the JB method requires a relatively large number of received samples for providing good detection performance. Since the CRs should detect PUs as soon as possible, the SF method would be preferred over JB-based SS.

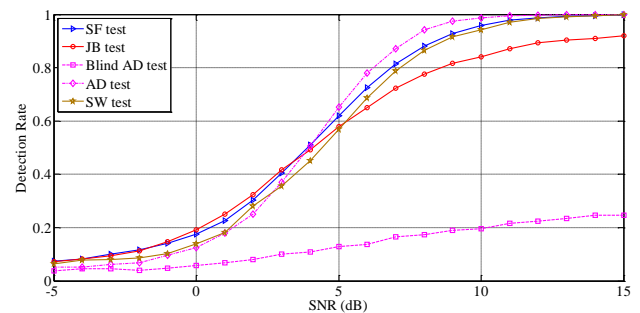


Fig. 5. Detection rate versus SNR value for simulated WLAN signal ( $N=800$ ) over AWGN channels ( $P_{fa}=0.05$ ).

Fig. 6 shows the detection performances over frequency-flat Rayleigh fading channels for  $N=800$ . As we can see, the Blind AD performance is very poor, while the mixed method performs better than the other methods and has a close performance comparing AD. Considering the acceptable performance of SF method, and the less complexity of SF, it has the best performance due to its less complexity and high detection probability.

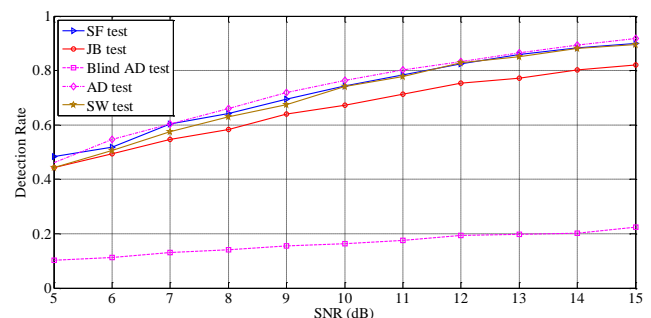


Fig. 6. Detection rate versus SNR value for the simulated WLAN signal ( $N=800$ ) over Rayleigh fading channels ( $P_{fa}=0.05$ ).

In Fig. 7, the performance of the proposed methods versus false alarm probability is shown. The ROC of the proposed methods for sample size 3200 (40 OFDM symbol) in WLAN signal is compared with AD method showing improvement in AD algorithm.

Considering Fig. 3 to Fig. 7, SF test has the best performance, so, we chose SF detector to compare with ED.

Fig. 7 shows the detection performance of SF test in comparison to Energy Detector in presence of uncertainty for  $P_{fa} = 5\%$  in colored WGN.

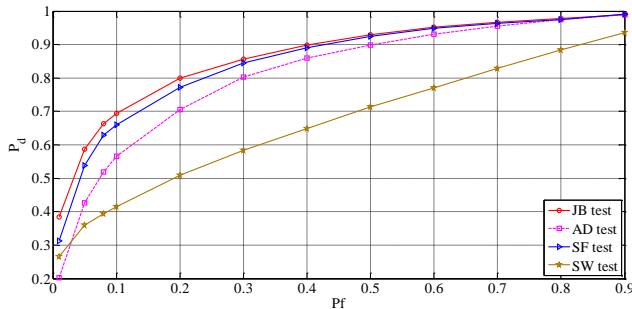


Fig. 7. ROC figure in WLAN for 3200 sample size (40 OFDM symbol) over frequency-flat Rayleigh fading channels with SNR = 0 dB.

Fig.8 demonstrates that correlation between noise sample does not affect performance of proposed GoF methods; because there is no any assumption in proposed methods about independent of noise samples. Results of Fig. 8 are verified by our assumptions about the lack of need for independent signal samples.

Usually uncertainty in noise variance exists and ranges about 1 to 2 dB [5]. As it is evident, the SF test works better than the ED against noise uncertainty. This figure demonstrates that correlation between noise sample does not affect performance of proposed GoF methods and lack of need for independent samples in our assumptions verifies results of Fig. 8.

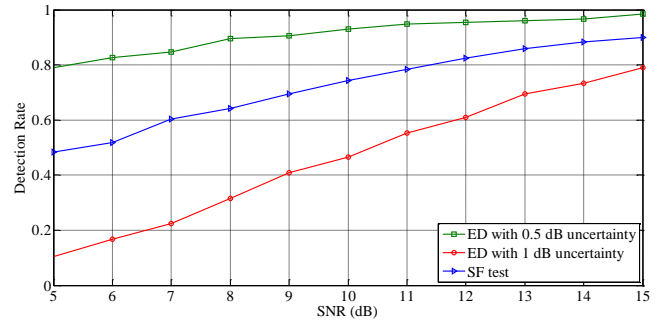


Fig. 8. Signal detection probability versus SNR for WLAN ( $N=800$ ) Additive colored Gaussian noise in the channel and over Rayleigh fading.

## 6. Conclusion

In this paper, three one-sample GoF techniques are introduced and modified to detect OFDM-based primary signals. The methods are compared through simulations with each other and a conventional approach (i.e. AD). We showed that the proposed methods' computational complexity is much less than the AD approach. Moreover, Monte-Carlo simulation results for OFDM-based primary signals demonstrate that the SF method outperforms the other techniques in terms of probability of detection. In particular, we have shown that the SF method performs well even in very short sensing durations, in comparison with other GoF-based sensing methods. Furthermore, simulation results show that the SF method outperforms the classical ED in presence of noise uncertainty over different level of SNR. Besides, it is not sensitive to the noise uncertainty which is favorable. Therefore, it can be concluded that the SF method is an appropriate representative of GoF tests in order to be applied in CR networks.



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**Seyed Sadra Kashef** was born in Urmia, Iran, in 1989. He received the M.Sc. degree in communication systems engineering from Tarbiat Modares University, Tehran, Iran, in 2011. He is currently pursuing the Ph.D. degree in communication systems engineering at Tarbiat Modares University. He is a student member of IEEE. His research interests are in the areas of statistical signal processing and spectrum sensing in cognitive radio networks.

**Paeiz Azmi** was born in Tehran-Iran, on April 17, 1974. He received the B.Sc., M.Sc., and Ph.D. degrees in electrical engineering from Sharif University of Technology (SUT), Tehran-Iran, in 1996, 1998, and 2002, respectively. Since September 2002, he has been with the Electrical and Computer Engineering Department of Tarbiat Modares University, Tehran-Iran, where he became a professor on December 2011. From 1999 to 2001, he

was with the Advanced Communication Science Research Laboratory, Iran Telecommunication Research Center (ITRC), Tehran, Iran. From 2002 to 2005, he was with the Signal Processing Research Group at ITRC. He is senior member of IEEE now.

**Hamed Sadeghi** was born in Tehran, Iran, in 1983. He received the M.Sc. degree in communication systems engineering from Tarbiat Modares University, Tehran, Iran, in 2008. He is currently pursuing the Ph.D. degree in communication systems engineering at Tarbiat Modares University. He is a student member of IEEE. His research interests are in the areas of statistical signal processing and its applications, including detection, estimation, data fusion, and spectrum sensing.