Parameter Estimation in Hysteretic Systems Based on Adaptive Least-Squares

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Abstract
In this paper, various identification methods based on least-squares technique to estimate the unknown parameters of structural systems with hysteresis are investigated. The Bouc-Wen model is used to describe the behavior of hysteretic nonlinear systems. The adaptive versions are based on the fixed and variable forgetting factor and the optimized version is based on optimized adaptive coefficient matrix. Simulation results show the efficient performance of the proposed technique in identification and tracking of hysteretic structural system parameters compared with other least square based algorithms.

Keywords: Hysteresis, Least-Squares Estimation (LSE), Optimization, System Identification.

1. Introduction

The System identification and fault detection based on measured vibration data through condition monitoring systems has been noticeable in recent years. Identifying the status of a structure and fault detection is the main goal of the condition monitoring systems for civil structures. Different methods of data analysis are reviewed in [1], which includes frequency domain analysis and time domain analysis methods. The advantages and shortcomings of damage identification methods are analyzed in [2]. Identification of structural systems can be categorized into two parts: online and offline. For online structural parameters changes identification, time domain analysis methods such as least-squares estimation [3-5] and filter-based methods such as Kalman filter [6,7], H_∞ filters [8], and wavelet technique [9] are used. Today, real-time detection of changes in structural parameters due to failures during events such as earthquakes is a challenging issue. Civil structures faced with intense earthquakes usually show hysteresis behavior. Various models have been proposed to identify and simulate the hysteresis, and Bouc-Wen model is the most appropriate [10]. This model is a quasi-physical and It can be used to describe the behavior of the wide range of considered systems [11,12].

Least-squares parameter estimation algorithm cannot estimate the time-varying parameters well. Adaptive LSE method to estimate time-varying parameters have been presented in [3,5]. A frequency domain nonlinear least-squares estimation algorithm was proposed in [13]. Fuzzy least-squares estimator was investigated in [14] by proposing a confidence region. A new structured total least-squares based frequency estimation algorithm for real sinusoids corrupted by white noise was adapted in [15].

In this paper, least-squares estimation algorithm and the adaptive versions based on the fixed and variable forgetting factor and optimized version to determine coefficients for tracking time-varying parameters are presented. Considered methods are applied for online identification of parameter changes of the nonlinear structural systems with hysteresis. The ability of the methods to track instantaneous changes in the parameters of a structural system due to failures is evaluated.

2. Problem Statement

Motion Equations of a *m* degrees of freedom can be described with Eq. (1).

$$M\ddot{x}(t) + F_c[\dot{x}(t)] + F_s[x(t)] = \eta f(t) \quad (1)$$

Where M is the mass matrix, $x(t)$ is the displacement vector, $F_C[\dot{x}(t)]$ is the dissipative force vector, $F_s[x(t)]$ is the non dissipative restoring force vector, $f(t)$ is the excitation vector, and η is the excitation influence matrix. Suppose we have a structure for estimating the unknown parameters including damping, stiffness, and hysteresis parameters; i.e., $[\theta_1(t), \theta_2(t), \ldots, \theta_n(t)]^T$. The observation equation associated with the motion equation of the structural system is expressed as

$$\varphi[\ddot{x}, \dot{x}, x; t]\theta(t) + \varepsilon(t) = y(t) \quad (2)$$

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where $\theta(t)$ is a vector of n unknown parameters. $\ddot{x}$, $\dot{x}$ and x are the measured acceleration, velocity and displacement response vectors. $y(t)$ and $\varepsilon(t)$ are the excitation and noise vectors. φ is a $m \times n$ data matrix. The Eq. (2) can be written at the time instant $t = t_k$ as

$$\varphi_k \theta_k + \varepsilon_k = y_k \tag{3}$$

The solution of the recursive LSE, $\hat{\theta}_{k+1}$ to estimate θ_{k+1} are given by the following equations [16].

$$\hat{\theta}_{k+1} = \hat{\theta}_k + K_{k+1}[y_{k+1} - \varphi_{k+1}\hat{\theta}_k]$$
$$K_{k+1} = P_k \varphi_{k+1}^T [I + \varphi_{k+1} P_k \varphi_{k+1}^T]^{-1}$$
$$P_k = [I - K_k \varphi_k] P_{k-1} \tag{4}$$

where K_{k+1} and $P_k^T = P_k > 0$ are the estimation and adaptation gain matrices.

3. Least-squares Estimation Based Algorithms

The Least-squares parameter estimation algorithm can not estimate the time-varying parameters well. A simple method is to use a fixed forgetting factor. In this method, P_k is replaced by $\bar{\lambda}^{-1} P_k$, where $\bar{\lambda} \in (0,1]$ is a constant forgetting factor. The ability to track parameters changes and the sensitivity to measurement noises increase in this method for a small forgetting factor. To enhance the performance a constant factor $\bar{\lambda}$ is replaced by $\bar{\lambda}_k$ in the variable forgetting factor method. Although in both methods when a parameter is changed other parameters are estimated with oscillations. To solve these problems and improve the real-time changes in parameter estimation the adaptive coefficient matrix method is proposed where each factor is adaptive to a specific changing parameter. The estimation error is corrected by a adaptive factor $\lambda_j(k+1)$ for estimating the parameter $\theta_j(k+1)$. Recursive solution for the vector of variable parameters is achieved as follows [5].

$$\hat{\theta}_{k+1} = \hat{\theta}_k + K_{k+1}[y_{k+1} - \varphi_{k+1}\hat{\theta}_k]$$
$$K_{k+1} = (\Lambda_{k+1} P_k \Lambda_{k+1}^T)\varphi_{k+1}^T [I + \varphi_{k+1}(\Lambda_{k+1} P_k \Lambda_{k+1}^T)\varphi_{k+1}^T]^{-1}$$
$$P_k = [I - K_k \varphi_k](\Lambda_k P_{k-1} \Lambda_k^T), \quad k = 1,2,\ldots \tag{5}$$

where $\Lambda_{k+1} = diag[\lambda_1(k+1), \lambda_2(k+1), \ldots, \lambda_n(k+1)]$ is a diagonal adaptive factor matrix.

The residual and predicted output error vectors $\bar{\gamma}_{k+1}$ and γ_{k+1} are defined as Eq. (6) and Eq. (7).

$$\bar{\gamma}_{k+1} = y_{k+1} - \varphi_{k+1}\hat{\theta}_{k+1} \tag{6}$$

$$\gamma_{k+1} = y_{k+1} - \varphi_{k+1}\hat{\theta}_k \tag{7}$$

Predicted output error covariance matrix is denoted by $V_{k+1} = E[\gamma_{k+1}\gamma_{k+1}^T]$, and when $\hat{\theta}_{k+1}$ reaches to θ_{k+1}. It would be

$$E[\bar{\gamma}_{k+1}\bar{\gamma}_{k+1}^T] = E[\varepsilon_{k+1}\varepsilon_{k+1}^T] = \sigma_{k+1}^2 \tag{8}$$

The adaptive tracking condition has been obtained as Eq. (9) follows in [5].

$$V_{k+1} - [I + \varphi_{k+1}(\Lambda_{k+1} P_k \Lambda_{k+1}^T)\varphi_{k+1}^T]\sigma_{k+1}^2$$
$$[I + \varphi_{k+1}(\Lambda_{k+1} P_k \Lambda_{k+1}^T)\varphi_{k+1}^T]^T = 0 \tag{9}$$

φ_{k+1} is measured and P_k, V_{k+1} and σ_{k+1}^2 are estimated. Eq. (9) is nonlinear, and it is difficult to find a solution. The objective function in Eq. (10) is used to find the optimal solution [5].

$$J[\hat{\theta}_{k+1}(\Lambda_{k+1})] = \sum_{j=1}^{n} \left| \frac{\hat{\theta}_j(k+1) - \hat{\theta}_j(k)}{\hat{\theta}_j(k)} \right| \tag{10}$$

Objective function (10) is the adding of the variation of parameters from $\hat{\theta}(k)$ to $\hat{\theta}(k+1)$. Finding the optimal solution Λ_{k+1} is a constrained optimization problem with the objective function (10) subject to the constraint of the Eq. (9) norm,

$$\| V_{k+1} - [I + \varphi_{k+1}(\Lambda_{k+1} P_k \Lambda_{k+1}^T)\varphi_{k+1}^T]\sigma_{k+1}^2$$
$$[I + \varphi_{k+1}(\Lambda_{k+1} P_k \Lambda_{k+1}^T)\varphi_{k+1}^T]^T \| \le \delta \tag{11}$$

The function "fmincon" in MATLAB is used to find an optimal solution for the adaptive factor matrix Λ_{k+1}. The initial value for Λ_{k+1} is supposed to be $\bar{\lambda}_{k+1}^{1/2} I$, where $\bar{\lambda}_{k+1} / [(V_{k+1}/\mu\sigma_{k+1}^2)^{1/2} - 1] / (\varphi_{k+1} P_k \varphi_{k+1}^T)$. If the calculated $\bar{\lambda}_{k+1}$ is smaller than one, it is set to be one. In this case all the parameters are constant at t_{k+1}.

4. Numerical Simulations

A nonlinear hysteretic structural system with one degree of freedom subject to earthquake acceleration $\ddot{x}_0(t)$ is considered.

$$m\ddot{x}(t) + r(\dot{x}, x) = -m\ddot{x}_0(t) \tag{12}$$

where x is the relative displacement and $r(\dot{x}, x) = F_c(\dot{x}) + F_s(x)$ is the total restoring force in which $F_c(\dot{x}) = c\dot{x}$. Bouc-Wen model is used to describe $r(\dot{x}, x)$ [12]:

$$\dot{r} = c\ddot{x} + k\dot{x} - \beta |\dot{x}| |r|^{n-1} r - \gamma |\dot{x}| |r|^n \tag{13}$$

where c is the damping coefficient, k is the equivalent stiffness, and β, n and γ are hysteresis parameters. Values of the parameters used in the simulation are shown in Table 1 ([3,5]). The El-Centro earthquake with a *5g* peak ground acceleration is used. For measured quantities the assumed sampling time is 1*KHz*.

To identify the parameters, Eq. (12) and Eq. (13) must be discrete and be converted to observation Eq. (3). The incremental component of restoring force r_k based on a 3rd order corrector method could be expressed as Eq. (14).

$$r_k - r_{k-1} = (\Delta t / 12)(5\dot{r}_k + 8\dot{r}_{k-1} - \dot{r}_{k-2}) \tag{14}$$

The unknown parameter vector is defined as a 4-vector $\theta_k = [c, k, \beta, \gamma]^T$. The me asured vector y_k can be computed from the measured data and is defined as Eq. (15).

$$y_k = (12 / \Delta t)(r_k - r_{k-1}) =$$
$$(-12m / \Delta t)(\ddot{x}_k - \ddot{x}_{k-1} + \ddot{x}_{0,k} - \ddot{x}_{0,k-1}) \tag{15}$$

The data matrix $\varphi_k = [\varphi_{k,1}\ \ \varphi_{k,2}\ \ \varphi_{k,3}\ \ \varphi_{k,4}]$ could be obtained as follows:

$$\begin{aligned}
&\varphi_{k,1} = 5\ddot{x}_k + 8\ddot{x}_{k-1} - \ddot{x}_{k-2}\\
&\varphi_{k,2} = 5\dot{x}_k + 8\dot{x}_{k-1} - \dot{x}_{k-2}\\
&\varphi_{k,3} = 5\,|\,\dot{x}_k\,||\,m(\ddot{x}_k + \ddot{x}_{0,k})\,|^{\alpha-1}\ m(\ddot{x}_k + \ddot{x}_{0,k})\\
&+8\,|\,\dot{x}_{k-1}\,||\,m(\ddot{x}_{k-1} + \ddot{x}_{0,k-1})\,|^{\alpha-1}\ m(\ddot{x}_{k-1} + \ddot{x}_{0,k-1})\\
&-|\,\dot{x}_{k-2}\,||\,m(\ddot{x}_{k-2} + \ddot{x}_{0,k-2})\,|^{\alpha-1}\ m(\ddot{x}_{k-2} + \ddot{x}_{0,k-2})\\
&\varphi_{k,4} = 5\dot{x}_k\,|\,m(\ddot{x}_k + \ddot{x}_{0,k})\,|^{\alpha} - 8\dot{x}_{k-1}\,|\,m(\ddot{x}_{k-1} + \ddot{x}_{0,k-1})\,|^{\alpha}\\
&+\dot{x}_{k-2}\,|\,m(\ddot{x}_{k-2} + \ddot{x}_{0,k-2})\,|^{\alpha}
\end{aligned} \tag{16}$$

Data matrix includes structural responses $\ddot{x}_k, \dot{x}_k$ and also the earthquake acceleration $\ddot{x}_{0,k}$. Seismic acceleration $\ddot{x}_{0,k}$ and acceleration response $\ddot{x}_k$ are measured using accelerometers. Velocity response $\dot{x}_k$ can be calculated using numerical integration of $\ddot{x}_k$. The hysteretic structural system parameters are shown in Table 1. The system stiffness k decreases suddenly at the moment of *t=15s* from *24.2kN/m* to *20kN/m* due to failure. Initial values were set to $c_0 = 0.1kNs/m$, $k_0 = 10kN/m$, $\beta_0 = 0$,$\gamma_0 = 0$,$P_0 = 100I$.

The identification results of a single degree of freedom structural system (SDOF) parameters using LSE algorithm, fixed and variable forgetting factor and the adaptive algorithm based on LSE and optimization are shown in Fig. 1 to Fig. 4. The time part *t<2* is not used for parameter identification, because the earthquake and its response are too small in this segment.

As shown in Fig. 1 the LSE algorithm can be used to identify fixed parameters, but this algorithm can not correctly identify the time-varying parameters. The constant forgetting factor which is used and shown in Fig. 2 modifies the variable parameter identification to a large extent, but other parameters are not well identified.

LSE with variable forgetting factor algorithm improves the results of the algorithm with a constant forgetting factor in Fig. 3. However, the identification of these two methods works regardless of which parameter changes and the oscillations seen in Fig. 2 and Fig. 3.

Parameter identification in adaptive algorithm based on LSE and optimized adaptive factor matrix is performed by setting the optimal coefficients and the identification error is very little in Fig. 4. Also the exact and identified hysteresis cycle with taking the stiffness variation in *t=15s* are shown in Fig. .

Table 1. Hysteretic system parameters

Parameter	Value
m, c, k β, γ, n	*125.53kg, 0.07kNs/ m, 24.2kN/ m 2, 1, 2*

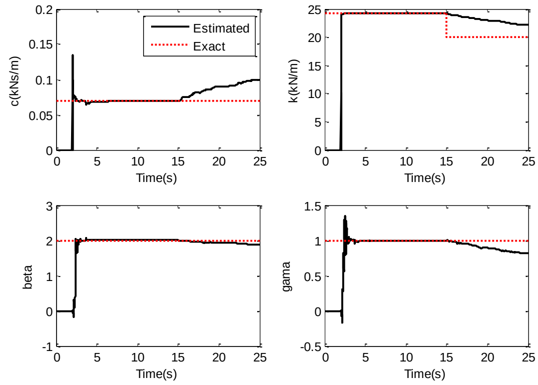

Fig. 1. Parameters c, k, β, γ identified using least-squares estimation algorithm.

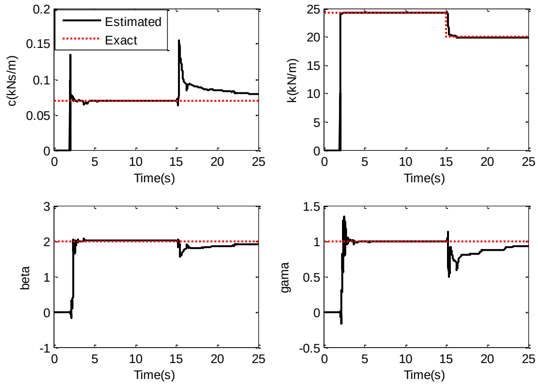

Fig. 2. Parameters c, k, β, γ identified using least-squares estimation algorithm with constant forgetting factor.

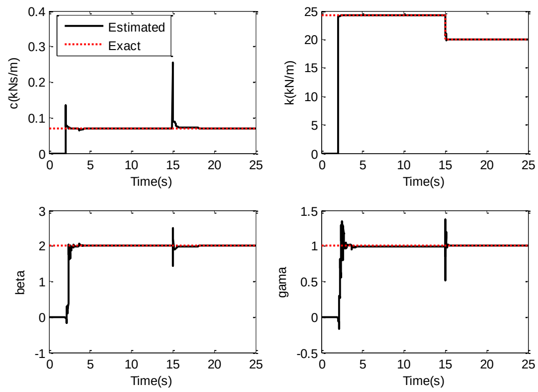

Fig. 3. Parameters c, k, β, γ identified using least-squares estimation algorithm with variable forgetting factor.

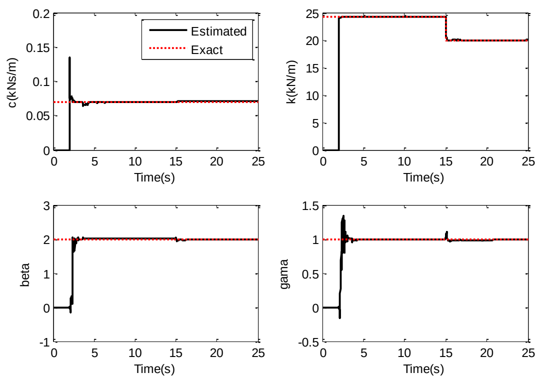

Fig. 4. Parameters c, k, β, γ identified using adaptive least-squares estimation algorithm with optimization.

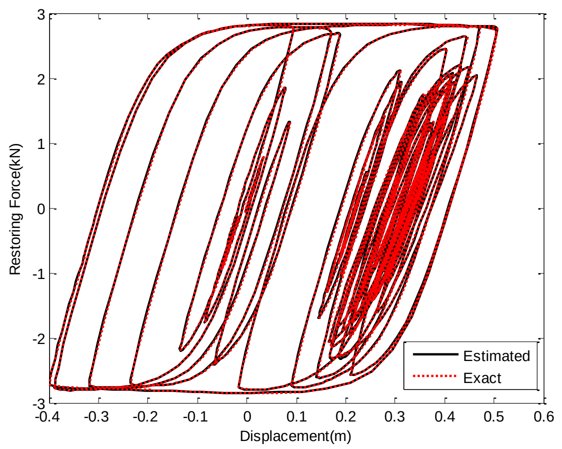

Fig. 5. Estimated and exact hysteresis cycles for a system with one degree of freedom with a stiffness loss.

5. Conclusions

Different identification algorithms based on least-squares estimation are used for identification of variable parameters of a hysteretic structural system. Efficiency of the adaptive coefficient matrix method is shown using the results of numerical simulations and compared with the results of ordinary least-squares estimation algorithms, fixed and variable forgetting factor algorithm. Results indicate that the adaptive coefficient matrix algorithm compared with other methods have better performance especially at the fault moment modeled as the immediate stiffness parameter change. Evaluation of the method to measurement noise, application to more complex structures and improvement of the optimization method are some future research topics.

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