Multiple Antenna Relay Beamforming for Wireless Peer to Peer Communications

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Abstract

This paper deals with optimal beamforming in wireless multiple-input-multiple-output (MIMO) relay networks that involves multiple concurrent source-destination pairs with imperfect channel state information (CSI) at the relays. Our aim is the optimization of the MIMO relay weights that minimize the total relay transmit power subject to signal-to-interference-plus-noise ratio (SINR) of all destinations to be kept above a certain threshold. Since power minimization is a non-convex quadratically constrained quadratic programming (QCQP), we use semi-definite programming (SDP) relaxation of above mentioned problem by using a randomization technique. Numerical Monte Carlo simulations verify the performance gain of our proposed multiple antenna relay system in terms of transmit power and symbol error probability.

Keywords: MIMO-Relay Networks, Power Allocation, Beamforming , Semi-Definite Programming

1. Introduction

Recently, using MIMO-relays in wireless networks has attracted significant attention. Due to the shadowing, multipath fading and interference, the link quality between the source and destination in a wireless network degrades intensively. Several schemes to achieve spatial diversity are considered in literature. The most popular cooperative schemes are amplify-and-forward (AF) [1], decode-and-forward (DF)[2], compress-and forward [3] and coded cooperation [4]. In the AF scheme, sources transmit messages to the relays, which then simply scale their received signals according to a power constraint and forward the scaled signals toward the destinations. AF scheme has received extensive attention due to its simplicity.

In [5], distributed beamforming relay system with single transmitter-receiver pair and several relaying nodes has been proposed and perfect CSI knowledge is supposed to be available for each node. The authors in [6] have investigated the same scenario as in [5], but have assumed that the second-order statistics of all channel coefficients are available at the receiver. In [6,7], the beamforming weights are obtained in order to maximize the signal-to-noise ratio (SNR) at the destination subject to individual relay power. In the subsequent scenario, the problem has been solved subject to the total relay power constraints, while [5] solved the beamforming weights only subject to individual relay power constraints. Although, both of them have the same problem formulation, they have completely different approaches to solving the problem.

MIMO systems attracted considerable attention because of their ability to support high data rate and wireless network improvement [8]. In this paper, we aim to design optimal beamforming in MIMO relay networks.

Conic optimization techniques, as a result of modern convex optimization, have been extensively used in [9] to obtain a computational attractive problem emerged from the original difficult problem. The optimization problem of [7,10,11] is shown to be nonconvex quadratically constrained quadratic program, which can be solved by relaxing the original problem SDP problem [12], and employing the interior point methods (IPM) [13] for solving the SDP problem. Its problem has been solved efficiently because their solutions have been rank-one.

The aim of this paper is the optimization of MIMO relay weights that minimize the total relay transmit power subject to SINR of all destinations be kept above a certain threshold. We show that such a power minimization problem is a non-convex QCQP problem. We turn it into a semi-definite programing problem using a well-known relaxation technique which is NP-hard in general, but in our case can be efficiently and exactly solved using a randomization technique.

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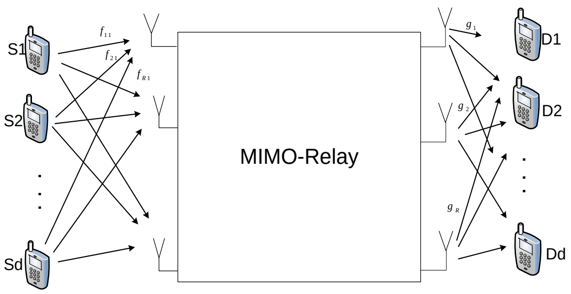

Figure 1: A MIMO-Relay Multiuser Network

2. System Model

We consider a peer to peer MIMO-relay network with d pairs of source-destination nodes as shown in Fig. 1. We have assumed that all source and destination nodes are equipped with one antenna and each source aims to maintain communication with its corresponding destination. All nodes work in half duplex mode and it is assumed that there is no direct link between source and destination. We use a two-step AF protocol. During the first step, each source broadcasts its signals to MIMO-relay. For the P_{th} user, given $\mathrm{s_p}$ as the source signal, the received signals at the MIMO-relay are collectively given as

$$\chi_r = \sum_{p=1}^{d} f_{rp} s_p + v'_r \in \mathbb{C}^{R\times 1}$$

where f_{rp} is the channel coefficient from the P_{th} source to the r_{th} MIMO-relay antenna, $\mathrm{s_p}$ is the information symbol assumed to be uncorrelated i.e.$E\{s_p s_q^*\} = P_p \delta_{pq}$ and v'_r represents the AWGN at the MIMO relay, whose components are i.i.d. zero mean circularly symmetric Gaussian noise with unit variance ,i.e., CN(0,1). For simplicity, we rewrite the vector notation of the received signals as

$$x = \sum_{p=1}^{d} \mathrm{f}_p\, s_p + \mathrm{V}' \quad (1)$$

where

$$X = [X_1, X_2, \ldots, X_R]^T,\ V' = [v_1, v_2, \ldots, v_R]^T$$

and $\mathrm{f}_p = [f_{1p}, f_{2p}, \ldots, f_{Rp}]^T$

At MIMO-relay, the received signal for P_{th} user is processed by a complex beamforming weights $\mathbf{W} \in \mathbb{C}^{R\times R}$, which should be designed appropriately. During the 2nd step, the adjusted signal retransmitted by MIMO-relay is

$$\mathbf{t} = \mathbf{W}^H x \quad (2)$$

where $\mathbf{t}$ is an $R \times 1$ vector whose r_{th} entry is the signal transmitted by the r_{th} MIMO-relay antenna. Let us denote the vector of the channel coefficient from the MIMO-relay to the k_{th} destination as $\boldsymbol{g}_k = [g_{1k}\ g_{2k} \ldots g_{Rk}]^T$ .The received signal at the k_{th} destination is expressed as

$$y_k = \mathbf{g}_k^T \mathbf{t} + n_k = \mathbf{g}_k^T \mathbf{W}^H \boldsymbol{x} + n_k$$

$$= \mathbf{g}_k^T \mathbf{W}^H \sum_{p=1}^{d} \mathbf{f}_{rp} \boldsymbol{s}_p + \mathbf{g}_k^T \mathbf{W}^H \mathbf{V}' + n_k$$

$$= \underbrace{\mathbf{g}_k^T \mathbf{W}^H \mathbf{f}_k s_k}_{\text{desired signal}} + \underbrace{\mathbf{g}_k^T \mathbf{W}^H \sum_{p=1, p=k}^{d} \mathbf{f}_{rp}\, \boldsymbol{s}_p}_{\text{Interfereence}} + \underbrace{\mathbf{g}_k^T \mathbf{W}^H \mathbf{V}' + n_k}_{\text{totalnoise}} \quad (3)$$

where n_k is the noise at the receiver, which is also assumed to be CN(0,1).

3. Optimal Relay Power Control

We aim to find the beamforming weights such that the MIMO-relay transmit power is minimized while maintaining the destinations QOS at a certain level, i.e., every destination SINR is required to be larger than a certain threshold value. The optimization problem is formulated as follows:

$$\underset{w}{\text{Minimize}}\ P_T$$
$$\text{Subject to}\ SINR_k \geq \gamma_k,\ for\ k = 1, 2, \ldots, d \quad (4)$$

where P_T is the MIMO-relay transmit power, $SINR_k$ and γ_k are the SINR and the target SINR at the k_{th} destination, respectively. Then, the SINR at the k_{th}destination is given by

$$SINR_k = \frac{P_s^k}{P_n^k + P_i^k} \quad (5)$$

Here, P_s^k , P_n^k and P_i^k represent the power of desired signal, noise and interference at the k_{th} destination, respectively.

We now derive the expressions for the total transmit power of the MIMO-relay and $SINR_k$. Using (2), the total MIMO-relay transmit power can be obtained as

$$P_T = E\{\mathbf{t}^H \mathbf{t}\} = E\{X^H \mathbf{W}\mathbf{W}^H X\} \quad (6)$$
$$= \text{trace}\{\mathbf{W}^H \mathbf{R}_x \mathbf{W}\}$$

where $\mathbf{R}_\mathrm{x} \triangleq E\{XX^H\}$ and can be expressed $\mathbf{R}_\mathrm{x}$ as

$$\mathbf{R}_x = \sum_{P=1}^{d} P_P\, E\{\mathbf{f}_p \mathbf{f}_p^H\} + \sigma_v^2 \mathbf{I}_{R\times R} \quad (7)$$

where $\mathbf{R}_f^p \triangleq E\{\mathbf{f}_p \mathbf{f}_p^H\}$ and trace$\{.\}$ represents the trace of a matrix.

Using Kronecker identity, we have

$$trace(\mathbf{A}\mathbf{X}^H \mathbf{B}\mathbf{X}) = Vec(\mathbf{X})^H (\mathbf{A}^T \otimes \mathbf{B}) Vec(\mathbf{X})$$

Then, we can rewrite the total transmit power of MIMO-Relay as

$$P_T = Vec(\mathbf{W})^H (\mathbf{I}_{R\times R} \otimes \mathbf{R}_x) Vec(\mathbf{W}) \quad (8)$$
$$= \mathbf{w}^H \mathbf{D}\mathbf{W}$$

Let us define $\mathbf{w} \triangleq Vec(\mathbf{W}) and \mathbf{D} \triangleq (\mathbf{I}_{R\times R} \otimes \mathbf{R}_x)$ where $Vec(.)$ is the vectorization operator which stacks all columns of a matrix on top of each other and $\otimes$

represents the kronecker multiplication of two matrices. Using desired signal component of (3), we can obtain P_s^k as

$$\begin{aligned} P_s^k &= E\left\{\mathbf{g}_k^T \mathbf{W}^H \mathbf{f}_k \mathbf{f}_k{}^H \mathbf{W} \mathbf{g}_k^*\right\} E\left\{|s_k|^2\right\} \\ &= Vec(\mathbf{W})^H \left(\mathbf{R}_{\mathbf{g}_k}{}^T \otimes \mathbf{R}_{f_k}\right) Vec(\mathbf{W}) \\ &= \mathbf{w}^H \mathbf{R}_k \mathbf{w} \end{aligned} \quad (9)$$

where, $\mathbf{R}_{gk} \triangleq E\{\mathbf{g}_k^* \mathbf{g}_k^T\}$, $\mathbf{R}_k \triangleq (\mathbf{R}_{gk}^T \otimes \mathbf{R}_{fk})$, and the total noise power is given by

$$\begin{aligned} P_n^k &= E\left\{v^H \mathbf{W} \mathbf{g}_k^* \mathbf{g}_k^T \mathbf{W}^H v\right\} + \sigma_n^2 \\ &= \sigma_v^2 \text{trace}\left\{\mathbf{W}^H \mathbf{W} \mathbf{R}_{\mathbf{g}_k}\right\} + \sigma_n^2 \\ &= \sigma_v^2 \left\{Vec(\mathbf{W})^H \left(\mathbf{R}_{\mathbf{g}_k}^T \otimes \mathbf{I}\right) Vec(\mathbf{W})\right\} + \sigma_n^2 \\ &= \mathbf{w}^H \mathbf{D}_k \mathbf{w} + \sigma_n^2 \end{aligned} \quad (10)$$

where $\mathbf{D}_k = \sigma_v^2 (\mathbf{R}_{\mathbf{g}t}^T \otimes \mathbf{I})$.

Denoting $\mathbf{D}_k = \{1,2,\dots,d\} - \{k\}$ and using the interference component of (3), the interference power at k_{th} destination can be obtained as

$$\begin{aligned} P_i^k &= E\left\{\mathbf{g}_k^T \mathbf{W}^H \left(\sum_{p,q \in D_k} \mathbf{f}_p \mathbf{f}_q^H s_p s_q^*\right) \mathbf{W} \mathbf{g}_k^*\right\} \\ &= trace\left\{\mathbf{W}^H \left(P_p \sum_{p \in \mathbf{D}_k} \mathbf{R}_{\mathbf{f}_p}\right) \mathbf{W} E\left\{\mathbf{g}_k^* \mathbf{g}_k^T\right\}\right\} \\ &= trace\left\{\mathbf{W}^H \mathbf{A}_k \mathbf{W} \mathbf{R}_{\mathbf{g}_k}\right\} = Vec(\mathbf{W})^H \left(\mathbf{R}_{\mathbf{g}_k}^T \otimes \mathbf{A}_k\right) Vec(\mathbf{W}) \\ &= \mathbf{w}^H \mathbf{Q}_k \mathbf{w} \end{aligned} \quad (11)$$

where

$$\mathbf{A}_k \triangleq \left(P_P \sum_{p=D_k} \mathbf{R}_{\mathbf{f}_p}\right), \mathbf{Q}_k \triangleq (\mathbf{R}_{\mathbf{g}t}^T \otimes \mathbf{A}_k)$$

Finally, the optimization problem can be written as:

$$\begin{aligned} &\underset{w}{\text{Minimize}}\ \mathbf{w}^H \mathbf{D} \mathbf{w} \\ &\text{Subject to } \frac{\mathbf{w}^H \mathbf{R}_k \mathbf{w}}{\mathbf{w}^H (\mathbf{D}_k + \mathbf{Q}_K) \mathbf{w} + \sigma_n^2} \geq \gamma_k \\ &\qquad for\ k = 1,2,\dots,d \end{aligned} \quad (12)$$

Since $\mathbf{w}^H(\mathbf{D}_k + \mathbf{Q}_K)\mathbf{w} + \sigma_n^2 \geq 0$, the above problem is equivalent to

$$\begin{aligned} &\underset{w}{\text{Minimize}}\ \mathbf{w}^H \mathbf{D} \mathbf{w} \\ &\text{Subject to } \mathbf{w}^H \left(\mathbf{R}_k - \gamma_k (\mathbf{D}_k + \mathbf{Q}_K)\right) \mathbf{w} \geq \gamma_k \sigma_n^2 \\ &\qquad for\ k = 1,2,\dots,d \end{aligned} \quad (13)$$

Let us define $\mathbf{T}_K \triangleq \mathbf{R}_k - \gamma_k(\mathbf{D}_k - \mathbf{Q}_k)$. In our QCQP problem, since all the matrices $\mathbf{T}_k$ are negative semidefinite for all k, the problem is convex and can be solved efficiently. However, the feasible set of our optimization problem is empty, since $\mathbf{w}^H \mathbf{T}_k \mathbf{w} \leq 0$ for all k and w. Therefore, the non-convex equality constraint (13) reveals that the QCQP problem is non-convex and NP-hard in general. However, we will show that an exact and simple solution can be found in our specific problem

4. Solution via SDP Relaxation

We first turned our QCQP problem into the semidefinite programing (SDP) problem. Let us define $\mathbf{X} \triangleq \mathbf{w}\mathbf{w}^H$, thus we recast the problem as follows:

$$\begin{aligned} &\underset{X}{\text{Minimize}}\ \ \text{trace}(\mathbf{D}\mathbf{X}) \\ &\text{Subject to } \text{trace}(\mathbf{T_k X}) \geq \gamma_k \sigma_n^2,\ for\ k = 1,2,\dots,d \\ &\qquad and\ Rank(\mathbf{X}) = 1, \mathbf{X} \geq 0 \end{aligned} \quad (14)$$

The rank-one constraint on **X** is nonconvex, hence the problem is nonconvex. By dropping the nonconvex rank-one constraint, the problem can be relaxed to a convex SDP problem. The relaxed version of the problem (14) can be represented by the following SDR[1] form. Now we can obtain a lower bound on the optimal value of (14) by solving this relaxed problem.

$$\begin{aligned} &\underset{\mathbf{X}}{\text{Minimize}}\ \text{trace}(\mathbf{D}\mathbf{X}) \\ &\text{Subject to } \text{trace}(\mathbf{T_k X}) \geq \gamma_k \sigma_{\varsigma_k}^2, \text{for } k = 1,2,\dots,d \\ &\qquad \mathbf{X} \geq 0 \end{aligned} \quad (15)$$

Modern SDP solvers, such as SeDuMi [14,15], use interior point methods to find an efficient optimal solution for the problem, if it be feasible; otherwise, they return to an assertion of infeasibility. Generally, the optimal value of SDP problem is a lower bound for the optimal value of the nonconvex QCQP problem, because the feasible set of problem (14) is only a subset of the feasible set of problem (15). If the optimal value in (15) ,i.e. $\mathbf{X_{opt}}$, is rank-one, then its principal eigenvector is exactly the optimal solution of the original optimization problem, otherwise, a rank-one solution for the original problem can be found using a randomization technique. The idea behind this technique is to generate candidate sets of beamforming vectors from the optimal solution matrix $\mathbf{X_{opt}}$ of problem (15) [16]. The accuracy of these techniques for semidefinite problem has been analyzed for different problems in [17,18] and it has been found that the randomization has acceptable performance in practical scenarios. In order to achieve this goal, first the eigenvalue decomposition of $\mathbf{X_{opt}}$ should be calculated as $\mathbf{X_{opt}} = VDV^H$. Then the candidate vector is generated as $x_c = VD^{(1/2)}P_c$, where P_c is a circularly symmetric complex white Gaussian vector generated as $P_c \in \mathbb{C}^{R\times 1} \sim Nc(0,1)$. Hence, it can be recognized that the vector x_c satisfies $E\{x_c x_c^H\} = \mathbf{X_{opt}}$. This random vector generation procedure should be performed multiple times and in each iteration, any vector (or its scaled version) that satisfies SINR constraints of problem (15) is saved as a candidate vector x'_c along with corresponding objective values. The vector generation should be performed for a predetermined number of times. The final minimum solution can be obtained by a simple minimization over the finite set of objective values as an approximate solution for the problem (14). One way to solve the problem (14) by x_c is to find a proper scaling factor

[1] Semi Definite Representation

$\sqrt{\beta} \geq 0$. Applying β to (15), the following problem will be attained

$$\underset{\mathbf{X}}{\text{Minimize}}\ \beta\,\text{trace}(\mathbf{DX})$$
$$\text{Subject to } \beta\,\text{trace}(\mathbf{T_k X}) \geq \gamma_k \sigma^2_{\varsigma_k},\ \text{for } k = 1,2,...,d$$
$$\text{Rank}(\mathbf{X}) = 1,\ \mathbf{X} \geq 0 \tag{16}$$

In the above algorithm, the acceptable scaling factors are those that $\beta\,\text{trace}(\mathbf{T}_k\mathbf{X}) \geq 0$. Thus, the maximum scaling factor should be selected as [10]:

$$\beta = \max_{k=1,...,d}\left\{\frac{\gamma_k \sigma^2_{\varsigma_k}}{\text{trace}(\mathbf{T}_k\mathbf{X})}\right\} \tag{17}$$

Consequently, the approximate solution of problem (14) is $\sqrt{\beta}x_c$. In our case, after an acceptable number of iterations (around 40 iterations), the solution of the randomization problem approached to its lower bound (the optimal value of relaxed problem). Therefore, $\mathbf{X_{opt}}$ is an acceptable and near optimal solution for the original nonconvex problem.

Another Suboptimal solution of the original problem (14) can be found by using penalty function in the objective part of the problem and converting the objective function into the difference of two convex functions [19] subject to current convex constraints. [20,21] have developed an effective nonsmooth optimization algorithm based on the sub-gradient of Rank one function.

5. Computational Complexity

The aim of this section is to analyze the computational complexity of related the MIMO relay systems used in practice. According to records, Nesterov and Nemirovskii [22] were the pioneers who extended interior-point methods from linear optimization to semi-definite optimization problem and built up the polynomial complexity of the algorithm, at least in theory.

In this part, we assess the computational complexity of a standard problem with only equality constraints and then we extend conclusions to our case with inequality and/or equality constraints. The standard form of SDP problem is defined as:

$$\underset{\mathbf{X}}{\text{Minimize}}\ \text{trace}(\mathbf{CX})$$
$$\text{Subject to}\ \text{trace}(\mathbf{A}_i\mathbf{X}) = b_i,\ \text{for i} = 1,...,\text{d}$$
$$\mathbf{X} \geq 0 \tag{18}$$

where $\mathbf{C}$ and $\mathbf{A}_i (i \leq i \leq d)$ are symmetric $n \times n$ matrices and $b \in R^d$. Thus, for such a problem the complexity order with large-update (or long-step) algorithm based on the primal dual SDP algorithm is

$$\text{O}\left(\sqrt{n}\log n \log(n/\varepsilon)\right) \tag{19}$$

where ε denotes the accuracy parameter of the algorithm, while this algorithm with small-update (or short-step) still has $O(\sqrt{n}\,log(n/\varepsilon))$ iterations bound [13].

Small-update IPMs are confined to unacceptably slow progress, while large-update IPMs are more efficient for faster progress in practice. Although, large-update IPMs perform much more efficiently and IPM algorithms are effective in practice, they often have somewhat worse complexity bounds.

The complexity order of solving the SDP problem in (15) is polynomial time. To evaluate the complexity of the original problem, the dimension parameter n should be specified.

Therefore, the dimensions of the matrices used in the objective and constraints of the problem (15) should be determined.

Regarding to Kronecker product of two matrices, if $\mathbf{A} \in \mathbb{C}^{n\times n}$ and $\mathbf{B} \in \mathbb{C}^{m\times m}$, then $\mathbf{A}\otimes\mathbf{B}$ will be a $nm \times nm$ matrix. According to the vectors definite in (8) and the sizes of $\mathbf{R}_x \in \mathbb{C}^{R\times R}$ and $\mathbf{I} \in \mathbb{C}^{R\times R}$, the dimension of $\mathbf{D}$ will be

$$\mathbf{D} \in \mathbb{C}^{R^2\times R^2}$$

The same approach can be used to find the size of $\mathbf{X}$ and $\mathbf{T}_k$, and as a result we have: $\mathbf{X}, \mathbf{T}_k \in \mathbb{C}^{R^2\times R^2}$. Consequently, the dimension of matrices $\mathbf{X}, \mathbf{T}_k, \mathbf{D}$ is R^2. It is notable that the constraints of our problem are not the same as the ones in the standard SDP problem. Therefore, they have to be modified to be similar to the standard format. In order to achieve this goal, the first step is to define y_i so that the inequality constraints of (15) change to equality relations.

$$\text{trace}(\mathbf{T}_i\mathbf{X}) = \gamma_i \sigma^2_{\varsigma_k} + y_i,\ \text{for i} = 1,...,\text{d}$$
$$\mathbf{X} \geq 0,\ y_i \geq 0$$

Next, a new variable $\hat{\mathbf{X}}$ should be defined in order to standardize the problem:

$$\hat{\mathbf{X}} \triangleq \begin{bmatrix} \mathbf{X} & \mathbf{0}_{R^2\times d} \\ \mathbf{0}_{d\times R^2} & \begin{bmatrix} y_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & y_d \end{bmatrix} \end{bmatrix}$$

As a result, the following standard form will be attained.

$$\underset{\mathbf{X}}{\text{Minimize}}\ \text{trace}(\hat{\mathbf{D}}\hat{\mathbf{X}})$$
$$\text{Subject to}\ \text{trace}(\hat{\mathbf{T}}_i\hat{\mathbf{X}}) = b_i,\ \text{for i} = 1,...,\text{d}$$
$$\hat{\mathbf{X}} \geq 0 \tag{20}$$

where $\hat{\mathbf{D}} \triangleq \begin{bmatrix} \mathbf{D} & \mathbf{0}_{R^2\times d} \\ \mathbf{0}_{d\times R^2} & \mathbf{0}_{d\times d} \end{bmatrix}$, $\hat{\mathbf{T}}_i \triangleq \begin{bmatrix} \mathbf{T}_i & \mathbf{0}_{R^2\times d} \\ \mathbf{0}_{d\times R^2} & \mathbf{0}_{d\times d} \end{bmatrix}$

Then, it is clear that

$$n = R^2 + d \approx R^2 \tag{21}$$

Consequently, the worst case complexity of solving (15) is $O(R\log(R^2)\log(R^2/\varepsilon))$ (22)

6. Numerical Results

In this section, we examine the performance of the proposed MIMO relay system in various scenarios. The channel vectors, **f** and **g** are assumed to be statistically independent and generated as i.i.d. complex Gaussian random variables with zero mean and different values of variances. Moreover, in all simulation results, the output power of all sources are assumed to be equal, i.e., $\{P_k\}_{k=1}^{d} = P = 0\text{dB}$, and also

$$\{\gamma_k\}_{k=1}^{d} = \gamma_{th}, \quad \{\sigma_v^2\}_{i=1}^{R} = \{\sigma_{\varsigma_k}^2\}_{k=1}^{d} = \sigma^2$$

Throughout our numerical examples, the noise power is normalized by the source transmit power, i.e. $\sigma^2 = 0dB$. Also, we assume that all of the channel coefficients are exactly known at a processing center in which the beamforming weights should be optimized.

In all of the cases, we solved the relaxed version of optimization problem using CVX software. If $\mathbf{X_{opt}}$ is rank-one matrix, its principal component is used to determine the solution of problem (14); otherwise the best rank-one approximation is obtained using the randomization procedure as described in section 4. In all of our scenarios, the relay transmit power is plotted for those values which qualify QoS constraint and each curve is plotted only for those threshold values for which the beamforming problem is feasible for at least 80% of the total realizations.

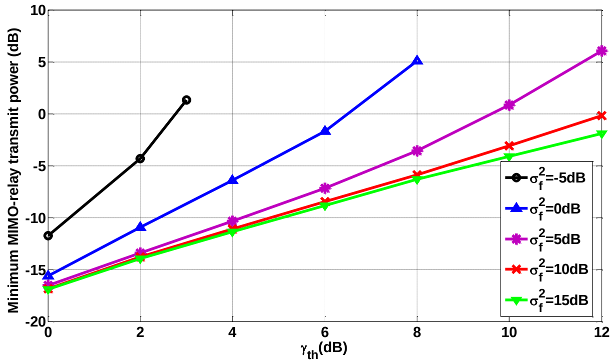

Figure 2: Minimum MIMO-relay transmit power P_T^{min} versus destination SINR threshold value γ_{th}, for different values of σ_f^2 and $\sigma_g^2 = 10\text{dB}$

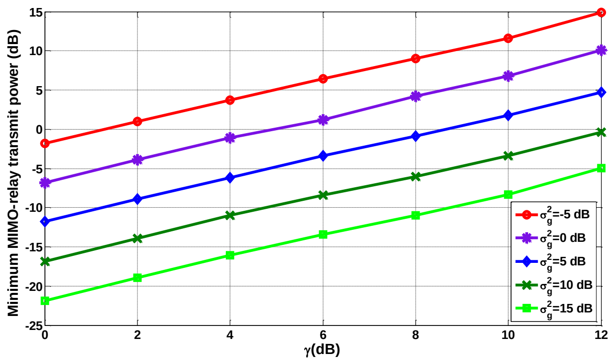

Figure 3: Minimum MIMO-relay transmit power P_T^{min} versus destination SINR threshold value γ_{th}, for different values of σ_g^2 and $\sigma_f^2 = 10\text{dB}$

σ_g^2 and σ_f^2 show the effect of the quality of the downlink and uplink channels, respectively. In Figure 2, we have plotted minimum MIMO-relay transmit power P_T^{min} versus destination SINR threshold value γ_{th}, for $\sigma_g^2 = 10dB$ and different values of σ_f^2. In this simulation setup, we have quantified the total relay transmit power by changing the SINR threshold value γ_{th}, from 0 dB to 25dB. The results are averaged over 1000 realizations of channel coefficients with CSG[1] distribution shown in Figure 2 for different values of σ_f^2. As can be seen from this figure, the better quality of uplink channel, the less minimum transmit power required to meet a certain QOS.

Moreover, as expected, this figure shows that the transmit power for all cases increases with increasing γ_{th}, due to the fact that, in order to achieve higher QOS requirement, the MIMO relay needs to spend more of its transmit power.

Figure 3 illustrates the minimum transmit power of MIMO-relay versus γ_{th}, for $\sigma_f^2 = 10dB$ and different values of σ_g^2. This figure also shows the effect of the quality of downlink channel on the minimum transmit power.

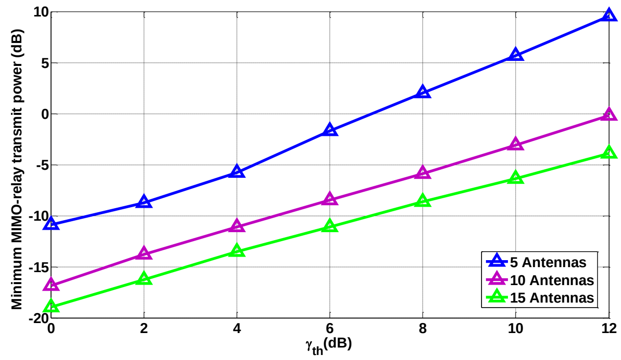

Figure 4: Minimum MIMO relay transmit power P_T^{min} versus destination SINR threshold value γ_{th}, for different number of antennas.

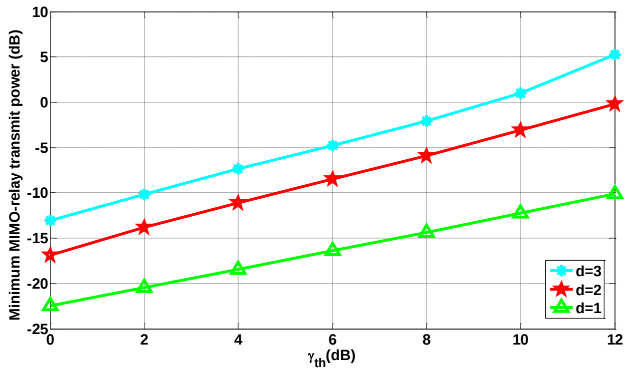

Figure 5: Minimum MIMO relay transmit power P_T^{min} versus destination SINR threshold value γ_{th}, for different number of source-destination pairs.

To study the effect of the number of MIMO-relay antennas in terms of quality of channels, we consider three different networks with different number of relay antennas, while setting $\sigma_f^2 = 10dB$, $\sigma_g^2 = 10dB$. Figure 4 illustrates the average minimum transmit power of MIMO-relay

[1] Complex Symmetric Gaussian

versus γ_{th} for two users and different number of antennas. As expected, it is observed that for a certain minimum value of received SINR, more power saving will be obtained by increasing the number of antennas. It can be seen that increasing the number of antennas from 5 to 10 results in at least 9 dB improvement in transmit power. This performance improvement decreases as the number of antennas increases.

In Figure 5, we investigate the performance of the network by changing the number of source-destination pairs. This figure reveals that the lower the number of user, the lower the minimum MIMO-relay transmit power required to meet a certain threshold level.

7. Conclusions

This paper has studied the optimal beamforming design in wireless multi-user MIMO-relay network to minimize MIMO-relay transmit power with guaranteed QOS at destinations. The proposed designs are based on a two-step amplify-and-forward protocol and imperfect channel state information. It has been shown that the corresponding optimization problem is nonconvex, but it can be converted into a convex problem by using a semidefinite relaxation technique and can be solved efficiently and accurately using well-known randomization technique.

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