A New Upper Bound for Free Space Optical Channel Capacity Using a Simple Mathematical in Equality

Arezu Rezazadeh*
Department of Electrical Engineering, Ferdowsi University of Mashhad
arezu.rezazadeh@gmail.com
Ghosheh Abed Hodtani
Department of Electrical Engineering, Ferdowsi University of Mashhad
ghodtani@gmail.com

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Abstract
In this paper, by using a simple mathematical inequality, we derive a $ new upper bound fkr the capacity of$ free space optical channel in coherent case. Then, by applying general fading distribution, we obtain an upper bound for mutual information in non-coherent case. Finally, we derive the corresponding optimal input distributions for both coherent and non-coherent cases, compare the results with previous works numerically and illustrate that our results subsume some of previous results in special cases.

Keywords: Mathematical Inequality, Capacity, Upper Bound, Free Space Optical Channel, Optimal Input Distributions.

1. Introduction

Free space optical (FSO) channel is important because of high transmission rate, power efficiency, high bandwidth and its safety.

To design communication link with high performance, it is necessary to study its properties from information theoretical viewpoint. To determine channel capacity, optimum input distribution should be obtained. By considering input constraints, the optimum input distribution is derived. In FSO channel, for eye safely and physical limitations, average and peak power constraints are imposed on transmitted signal [1]. The mathematical representation for FSO channel is [1]:

$$Y = HX + Z, \tag{1}$$

Where, X is the channel input, *Y* is the output and *Z* is the Gaussian noise with zero mean and variance of *σ2* or *Z ~ N(0;σ2)*. H represents channel fading which has the probability density function *f(h)*. The input constraints are [1]:

$$0 \le X \le A,\ \ E\{X\} \le P,\ \ \rho = \frac{A}{P}, \tag{2}$$

Where A is the peak-amplitude limit and *P* is the average power limit and *ρ* is the ratio of optical peak to average power.

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The authors are with the Department of Electrical Engineering, Ferdowsi University of Mashhad, Mashhad, Iran (e-mail: arezu.rezazadeh@gmail.com; ghodtani@gmail.com).

Previous Works: In [2] with constraints on input amplitude and power, it was shown that in coherent receiver, the capacity-achieving input distribution is discrete with a finite number of mass points. In other words, the input maximizing *I(X; Y| h)* is:

$$P = \{p_x(x) : p_x(x) = \sum_{i=0}^{K} a_i \delta(x - x_i), x_i \in [\circ, A],$$
$$a_i \ge 0, \sum_{i=\circ}^{K} a_i = 1, K \in Z^+, P \ge \sum_{i=\circ}^{K} x_i a_i\}, \tag{3}$$

Where *δ(x)* is the delta function and *Z+* is the set of positive integers. The number of mass points is *K + 1*, where ai and *xi* are the amplitudes and positions of the ith mass points, respectively [1], [2].

In [1] instead of maximizing mutual information, source entropy is maximized for the capacity of FSO channel.

In [3] under non-negativity and average optical power constraints lower and upper bounds for *I(X; Y|h = 1)* are derived. The lower and upper bounds are derived by maximizing source entropy and using a sphere packing argument respectively.

In [4] bounds for *I(X;Y|h = 1)* are derived by using a dual minimax problem (instead of maximizing the mutual information over distributions on the channel input alphabet, average relative entropy is minimized over

* Corresponding Author

distributions on the channel output alphabet). At high-power regime, a lower bound for *I(X; Y|h = 1)* is also proposed by using the entropy power inequality.

In [5] by considering Gaussian maximum entropy for *H(Y|h)*, an upper bound for *I(X; Y|h)* has been derived. Then by averaging over the gamma-gamma atmospheric turbulence for h, an upper bound for *I(X;Y)* (non-coherent case) has been determined.

Our Work: In this paper, we derive a new upper bound for the capacity of FSO channel in both coherent and non-coherent cases and determine the corresponding optimum input distributions for these two cases.

As pointed before, for additive noise with input peak and power constraints, the optimum input distribution is discrete with finite number of mass points [2]. Similarly for coherent case with these constraints, capacity achieving distribution is discrete with finite number of mass points. By considering this fact and using simple mathematical inequality, we determine a new upper bound for capacity of FSO channel. Then we extend the result to the non-coherent case with arbitrary *f(h)* and finally we determine the corresponding input distribution and compare the results with previous works.

Paper Organization: This paper has four sections. In section II, an upper bound for *I(X;Y|h)* and the corresponding input distribution is found. In section III, an upper bound for *I(X; Y)* (non coherent case) is derived by averaging over distribution of *f(h)*. Then we will maximize the upper bound of *I(X;Y)* over all input distributions. The paper concludes in section IV.

2. An Upper Bound for I(X; Y|h) and the Corresponding Input Distribution

In this section, first we determine an upper bound for *I(X;Y|h)* and then determine the corresponding input distribution. For discrete-time Gaussian channels [6], capacity can be expressed as:

$$C = \max_{f_x(x)} I(X;Y) = \max_{f_x(x)} \int I(X;Y\mid h) f(h) dh, \quad (4)$$

To reach *I(X;Y)*, we simplify *I(X;Y|h)*. *X* and *H* are independent, thus the mutual information, between channel input and output is [1]:

$$I(X;Y \mid H=h) = H(Y\mid H=h) - H(Y\mid X, H=h)$$
$$= -\int f(y\mid h)\log_2 f(y\mid h)dy + \int f(y\mid h,x)\log_2 f(y\mid h,x)dy \quad (5)$$

Where, in view of (1):

$$(y\mid h,x) \sim N(hx,\sigma^2) \quad (6)$$

$$f(y\mid h) = \int f_x(x) f(y\mid h,x)dx \quad (7)$$

Where *N(μ,σ2)* denotes a Gaussian distribution with mean μ and variance *σ2* and *fx(x)* is the input distribution in (3). Therefore,

$$f(y\mid h) = \int f_X(x) f(y\mid h,x)dx$$
$$= \int \sum_{i=0}^{K} a_i \delta(x - x_i) \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(y-hx)^2}{2\sigma^2}} dx$$
$$= \sum_{i=0}^{K} a_i \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(y-hx_i)^2}{2\sigma^2}}.$$

So,

$$I(X;Y\mid h) = -\int f(y\mid h)\log_2(f(y\mid h))dy - H(z) =$$
$$-\int \sum_{i=0}^{K} a_i \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(y-hx_i)^2}{2\sigma^2}} \log_2\Big(\sum_{j=0}^{K} a_j \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(y-hx_j)^2}{2\sigma^2}}\Big)dy$$
$$-H(z). \quad (8)$$

Since the above integral cannot be evaluated analytically, we will determine an upper bound for *I(X;Y|h).*

A. Upper Bound for I(X;Y|h)

In order to find an upper bound for *I(X;Y|h)*, we write *I(X;Y|h)* in terms of ais. From (8) we have:

$$I(X;Y\mid H=h) = -H(z)$$
$$-\int \sum_{i=0}^{K} a_i \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(y-hx_i)^2}{2\sigma^2}} \log_2\Big(\sum_{j=0}^{K} a_j \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(y-hx_j)^2}{2\sigma^2}}\Big)dy$$
$$= -H(z) - (\log_2 e).\Bigg[\int \sum_{i=0}^{K} a_i \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(y-hx_i)^2}{2\sigma^2}} \ln\Big(\frac{1}{\sqrt{2\pi\sigma^2}}\Big)dy$$
$$+\int \sum_{i=0}^{K} a_i \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(y-hx_i)^2}{2\sigma^2}} \ln\Big(\sum_{j=0}^{K} a_j e^{-\frac{(y-hx_j)^2}{2\sigma^2}}\Big)dy\Bigg]$$
$$\overset{a}{=} -H(z) - \log_2\Big(\frac{1}{\sqrt{2\pi\sigma^2}}\Big)$$
$$-(\log_2 e).\Bigg[\int \sum_{i=0}^{K} a_i \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(y-hx_i)^2}{2\sigma^2}} \ln\Big(\sum_{j=0}^{K} a_j e^{-\frac{(y-hx_j)^2}{2\sigma^2}}\Big)dy\Bigg] \quad (9)$$

where, a in (9) follows from the fact that $\sum_{i=0}^{K} a_i = 1$ and $\int \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(y-hx_i)^2}{2\sigma^2}} dy=1$.

Because ais are less than one, so $a_i e^{-\frac{(y-hx_i)^2}{2\sigma^2}}$ is less than 1. Furthermore

$$u_i \le 1 \Rightarrow \sum u_i \ge \prod u_i \quad \Rightarrow -\log \sum u_i \le -\log \prod u_i$$

hence,

$$-\ln(\sum_{i=0}^{K} a_i e^{-\frac{(y-hx_i)^2}{2\sigma^2}}) \le -\ln(\prod_{i=0}^{K} a_i e^{-\frac{(y-hx_i)^2}{2\sigma^2}}). \tag{10}$$

Now, we can determine an upper bound for *I(X;Y|h)*. From (9) and (10), the following upper bound is obtained.

$$I(X;Y \mid H=h) \le A_1 + A_2 h^2 \tag{11}$$

Where,

$$A_1 = -\log_2(\frac{1}{\sqrt{2\pi\sigma^2}}) - H(z) - \log_2(\prod_{j=0}^{K} a_j) + \frac{K+1}{2}\log_2^e, \tag{12}$$

And

$$A_2 = \log_2^e \sum_{n=0}^{K}\sum_{m=0}^{K} \frac{a_n}{2\sigma^2}(x_m - x_n)^2. \tag{13}$$

Now we should determine the corresponding input distribution.

B. Determining Optimum input distribution for upper bound of I(X;Y|h)

Here, we determine ais such that the upper bound in (11), regarding the constraints, becomes maximum. We use Lagrangian coefficients to determine optimum input distribution.

$$J_1 = A_1 + A_2 h^2 + \lambda_1^1(\sum a_i - 1) + \lambda_2^1(\sum a_i x_i - P). \tag{14}$$

To solve the optimization problem, considering constraints $\sum_{i=0}^{K} a_i = 1$ and $\sum_{i=0}^{K} a_i li = P$, we make $\frac{\partial J_1}{\partial a_i} = 0$. For subset of input distribution with $K + 1$ equally spaced mass points i.e., $xi = il$, where, $l = \frac{A}{K}$, we have:

$$a_i = \frac{1}{B_1 + B_2} \tag{15}$$

Where,

$$B_1 = (K+1) + \frac{h^2}{2\sigma^2} lK(K+1)P + \frac{h^2}{2\sigma^2}\left\{-(K+1)\sum_{j=0}^{K} a_j j^2 l^2 - K(K+1)l^2 i + i^2 l^2 (K+1)\right\},$$

and

$$B_2 = \frac{(il-P)}{(\sum_{j=0}^{K} a_j (jl)^2 - P^2)}\left[l\left(\frac{K(K+1)}{2}\right) - (K+1)P - \frac{h^2}{2\sigma^2}\left\{ lK(K+1)P^2 + (K+1)\sum_{j=0}^{K} a_j (lj)^3 - \left[(K+1)P + K(K+1)l\right]\sum_{j=0}^{K} a_j j^2 l^2 \right\}\right].$$

Optimum input distribution which maximizes the upper bound in (11) is derived via (15). It is clear that (15) is non linear and should be determined numerically. In general, optimal input distributions are different for each A (peak amplitude limit), *σ2* (variance of noise) and h. So for a given *A/σ*, h and ρ, optimal input distribution is determined numerically. By considering *h = 1*, *A = 1* and *σ=1* amplitude of mass points for, *ρ= 10* and *ρ = 2.5* are presented in Tables I and II respectively.

In coherent case by applying *h = 1*, to (15) we compare our derived upper bound with bounds which are derived in [4]. For a given *A/σ* and *ρ*, amplitude of mass points are computed for several K (number of mass points), and the corresponding upper bounds, which are derived from (11), are collected in a collection. The optimum number of mass points correspond to the upper bound which has minimum distance with lower bound. Fig. 1 illustrates the comparison between our upper bound (11) and bounds derived in [4]. At low *A/σ*, our upper bound is showing tighter performance than upper bounds which are proposed in [4]. Although at high *A/σ* there is a great gap between upper bounds derived from (11) and lower bound derived in [4], but our proposed upper bound is determined simply. The coherence time, for FSO channel is on the order of 1-100 msec [1]. To plot figure, we consider the coherence time 1 msec.

TABLE I: OPTIMAL INPUT DISTRIBUTION FOR COHERENT CASE (15), WHEN h = 1, ρ= 10 AND A/σ = 0dB

Number of mass points	a0	a1	a2	a3	a4
K=1	0.9	0.1			
K=2	0.8505	0.0989	0.0505		
K=3	0.8181	0.0975	0.0507	0.0337	
K=4	0.794	0.0964	0.0505	0.0339	0.0253

TABLE II: OPTIMAL INPUT DISTRIBUTION FOR COHERENT CASE

(15),WHEN h = 1,ρ= 2.5 AND A/σ = 0dB

Number of mass points	a0	a1	a2	a3	a4
K=1	0.6	0.4			
K=2	0.4307	0.3386	0.2307		
K=3	0.3409	0.2808	0.2158	0.1626	
K=4	0.2836	0.2398	0.1950	0.1563	0.1253

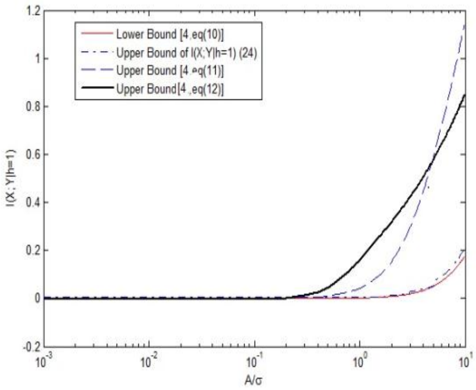

Fig. 1. Comparison of upper and lower bounds at low A/σ when h = 1 and ρ= 10.

3. An Upper bound for I(X; Y) and the corresponding input distribution

We want to compute $I(X;Y) = \int I(X;Y\,|h) f(h)dh$ and then maximize I(X;Y*)* over all input distributions. First we describe *f(h)* in terms of hyper-geometric functions and then continue aiming at finding the upper bound.

Description of *f(h)* in Terms of Hyper-geometric Functions

In FSO channel, the channel state *h* is the product of $g_a\, h_a\, h_p$, where g_a is the deterministic path loss, h_a is the random attenuation due to atmospheric turbulence and well modeled by a Gamma-Gamma distribution, and hp is the random attenuation due to geometric spread and pointing errors [1], [7], [8]. The probability density of *h* i.e., *f(h)* in [1] and [7] is expressed as:

$$f_h(h) = \frac{\gamma^2 h^{\gamma^2-1}}{(A_0 g_a)^{\gamma^2}} \int_{h/A_0 g_a}^{\infty} h_a^{-\gamma^2} f_{ha}(h_a) dh_a, \quad (16)$$

Where,

$$f_{h_a}(h_a) = \frac{2(\alpha\beta)^{\frac{\alpha+\beta}{2}}}{\Gamma(\alpha)\Gamma(\beta)} (h_a)^{\frac{\alpha+\beta}{2}-1} K_{\alpha-\beta}(2\sqrt{\alpha\beta h_a}), \quad (17)$$

Where Kα-*β(.)* is the modified Bessel function of the second kind, Γ (·) is the gamma function, and $1/\alpha$ and $1/\beta$ are the variances of small and large scale eddies respectively [1], and an expression for ga, γ and A0 is given in [1], [7]. A closed form for probability density function of *h* in terms of hyper-geometric functions, was computed in [8]. Considering $\Gamma(s)\Gamma(1-s) = \pi \csc(\pi s)$, the probability density of *h* [8,eq. (13)] can be expressed as:

$$f_h(h) = \frac{\gamma^2 h^{-1}}{\Gamma(\alpha)\Gamma(\beta)} \Bigg\{ \left(\frac{\alpha\beta h}{A_0 g_a}\right)^{\gamma^2} \Gamma(\beta-\gamma^2)\Gamma(\alpha-\gamma^2)$$
$$+ \frac{\left(\frac{\alpha\beta h}{A_0 g_a}\right)^{\alpha} \Gamma(\beta-\alpha)}{(\gamma^2-\alpha)} \times$$
$${}_1F_2\left(\alpha-\gamma^2; \alpha-\beta+1, \alpha-\gamma^2+1; \frac{\alpha\beta h}{A_0 g_a}\right)$$
$$+ \frac{\left(\frac{\alpha\beta h}{A_0 g_a}\right)^{\beta} \Gamma(\alpha-\beta)}{(\gamma^2-\beta)} \times$$
$${}_1F_2\left(\beta-\gamma^2; \beta-\alpha+1, \beta-\gamma^2+1; \frac{\alpha\beta h}{A_0 g_a}\right) \Bigg\} \quad (18)$$

Where 1F2(a;b,c;z) is a generalized hyper-geometric function with series representation:

$${}_1F_2(a;b,c;z) = \sum_{k=0}^{\infty} \frac{(a)_k}{(b)_k (c)_k} \frac{z^k}{k!}$$

Here (.)k represents the Pochhammer symbol, which is defined by

$(z)_0 = 1$ and $(z)_n = z(z+1)(z+2)...(z+n-1) = \Gamma(z+n)/\Gamma(z)$.

We expressed *f(h)*. Now, we can determine an upper bound for *I(X;Y)*.

Maximizing the Mutual Information and Determining an Expression for the Input Distribution

First we compute the following expression, then we determine ais, (with considering constraints) such that the upper bound of *I(X;Y)* will be maximized.

$$I(X;Y) \le \int (A_1 + A_2 h^2) f(h) dh. \quad (19)$$

We know that [9]:

$$\int_0^{K_c} h^a \; {}_1F_2(a_1; a_2, a_3; bh) dh = \frac{K_c^{\,a+1} \; {}_2F_3(a_1, a+1; a_2, a_3, a+2; bK_c)}{a+1} \quad (20)$$

Where,

$$ {}_2F_3(a_1,a_2;b_1,b_2,b_3;z)=\sum_{k=0}^{\infty}\frac{(a_1)_k\,(a_2)_k}{(b_1)_k\,(b_2)_k\,(b_3)_k}\frac{z^k}{k\,!} \quad (21)$$

So we can write:

$$ I(X;Y)\le\int (A_1+A_2h^2)f(h)dh= $$
$$ =\int_0^{K_c}A_1f(h)dh+\int_0^{K_c}A_2h^2f(h)dh. $$

Notice that, the integral of expectation, is defined from zero to constant K_c. It means that

$$ 0\le h\le K_c $$

We will see the dependence of mass points on this parameter (K_c) later. Considering (18) and (20), the upper bound for I(X; Y) can be expressed as:

$$ I(X;Y)\le A_1I_1+A_2I_2 \quad (22)$$

where

$$ I_1=\frac{\gamma^2}{\Gamma(\alpha)\Gamma(\beta)}[I_{1a}+I_{1b}+I_{1c}] \quad (23)$$

And

$$ I_2=\frac{\gamma^2}{\Gamma(\alpha)\Gamma(\beta)}[I_{2a}+I_{2b}+I_{2c}] \quad (24)$$

Where,

$$ I_{1a}=(\frac{\alpha\beta}{A_0g_a})^{\gamma^2}\Gamma(\beta-\gamma^2)\Gamma(\alpha-\gamma^2)\frac{K_c^{\gamma^2}}{\gamma^2} $$

$$ I_{1b}=\frac{(\frac{\alpha\beta}{A_0g_a})^{\beta}\Gamma(\alpha-\beta)}{(\gamma^2-\beta)}\times\frac{K_c^{\beta}\ {}_2F_3(\beta-\gamma^2,\beta;\beta-\alpha+1,\beta-\gamma^2+1,\beta+1;\frac{\alpha\beta}{A_0g_a}K_c)}{\beta} $$

$$ I_{1c}=\frac{(\frac{\alpha\beta}{A_0g_a})^{\alpha}\Gamma(\beta-\alpha)}{(\gamma^2-\alpha)}\times\frac{K_c^{\alpha}\ {}_2F_3(\alpha-\gamma^2,\alpha;\alpha-\beta+1,\alpha-\gamma^2+1,\alpha+1;\frac{\alpha\beta K_c}{A_0g_a})}{\alpha} $$

And

$$ I_{2a}=(\frac{\alpha\beta}{A_0g_a})^{\gamma^2}\Gamma(\beta-\gamma^2)\Gamma(\alpha-\gamma^2)\frac{K_c^{\gamma^2+2}}{\gamma^2+2} $$

$$ I_{2b}=\frac{(\frac{\alpha\beta}{A_0g_a})^{\beta}\Gamma(\alpha-\beta)}{(\gamma^2-\beta)}\times\frac{K_c^{\beta+2}\ {}_2F_3(\beta-\gamma^2,\beta+2;\beta-\alpha+1,\beta-\gamma^2+1,\beta+3;\frac{\alpha\beta K_c}{A_0g_a})}{\beta+2} $$

$$ I_{2c}=\frac{(\frac{\alpha\beta}{A_0g_a})^{\alpha}\Gamma(\beta-\alpha)}{(\gamma^2-\alpha)}\times\frac{K_c^{\alpha+2}\ {}_2F_3(\alpha-\gamma^2,\alpha+2;\alpha-\beta+1,\alpha-\gamma^2+1,\alpha+3;\frac{\alpha\beta K_c}{A_0g_a})}{\alpha+2} $$

and 2F3 has been defined in (21). Now, we maximize the upper bound of I(X; Y) over all input distributions and derive an expression for the input.

Determining Optimal input Distribution which Maximizes Our Upper Bound of *I(X; Y)*

We should determine a_is such that the upper bound in (22), regarding the constraints, becomes maximum. We define *J* as the Lagrangian associated with the optimization problem. Again similar to previous section, to solve the optimization problem, considering constraints $\sum_{i=0}^{K}a_i=1$ and $\sum_{i=0}^{K}a_i li=P$, we make $\frac{\partial J}{\partial a_i}=0$. For subset of input distribution with $K+1$ equally spaced mass points i.e., $xi = il$, where, $l=\frac{A}{K}$ we have:

$$ J=A_1I_1+A_2I_2+\lambda_1(\sum a_i-1)+\lambda_2(\sum a_i li-P). $$

by considering constraints $\sum_{i=0}^{K}a_i=1$ and $\sum_{i=0}^{K}a_i li=P$, the optimized *ais* which maximize the upper bound of *I(X; Y)*, can be expressed as:

$$ a_i=\frac{1}{D_1+D_2} \quad (25)$$

Where,

$$ D_1=(K+1)I_1+\frac{I_2}{2\sigma^2}lK(K+1)P $$
$$ +\frac{I_2}{2\sigma^2}\left(-(K+1)\sum_{j=0}^{K}a_j j^2l^2-K(K+1)l^2i+i^2l^2(K+1)\right), $$

and

$$D_2 = \frac{(il - P)}{\left(\sum_{j=0}^{K} a_j (jl)^2 - P^2\right)} \left[I_1 l \left(\frac{K(K+1)}{2} \right) - (K+1) I_1 P \right.$$

$$- \frac{I_2}{2\sigma^2} \left\{ lK(K+1)P^2 + (K+1) \sum_{j=0}^{K} a_j (lj)^3 \right.$$

$$\left.\left. - \left[(K+1)P + K(K+1)l \right] \sum_{j=0}^{K} a_j j^2 l^2 \right] \right\}.$$

So, the optimal input distribution, which maximizes the upper bound of *I(X; Y)*, has the above relation.

It is clear that, (25) is non linear and depends on channel parameters. Notice that neither the correct

Number of mass points (K) nor the values of them

(a_i) are known. The equation (25) is non linear and depends on the channel parameters. We should determine channel parameters, to compute a_i*s*. But due to complexity of equation (25), the numerical

Calculation have been done just for K = 1. It can be seen easily that, when K = 1, a_i*s* just depend on *ρ*. So it is clear that they are independent on Kc, which is the upper limit of integral in computing expectation of *I(X; Y |h)*, and other channel parameters. Thus, for K = 1, there is no need to know channel parameters. When K = 1, a_i*s* are determined as a function of ρ. By using (25) and (2), we have:

$$K = 1 \Rightarrow P = \sum_{i=0}^{1} a_i li = a_1 l$$

$$\Rightarrow a_1 = \frac{1}{\rho}, \qquad a_0 = \frac{\rho - 1}{\rho} \tag{26}$$

It is the exact result, given in [1]. Farid and Hranilovic have shown that, for K = 1, the amplitude of mass points are given by the following

Equation [1]:

$$[P_0, P_1] = [\frac{\rho - 1}{\rho}, \frac{1}{\rho}].$$

For K = 1, the amplitude of mass points, for coherent and non coherent, are the same and determined from (26).

4. Conclusion

In this paper, by using a simple mathematical inequality, we determined new upper bounds for capacity of FSO channel in coherent and non coherent cases. For *h* = 1 we compare our results with previous works. At low SNR our upper bound shows tighter performance. For non coherent case, optimum input distribution depends on channel parameters, but for two mass points, optimum value of mass points are independent of channel parameters and just depend on ρ. Our results subsume some of the previous ones in special cases.

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