# A Novel Resource Allocation Algorithm for Heterogeneous Cooperative Cognitive Radio Networks

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**Abstract**

In cognitive radio networks (CRN), resources available for use are usually very limited. This is generally because of the tight constraints by which the CRN operate. Of all the constraints, the most critical one is the level of permissible interference to the primary users (PUs). Attempts to mitigate the limiting effects of this constraint, thus achieving higher productivity is a current research focus and in this work, cooperative diversity is investigated as a promising solution for this problem. Cooperative diversity has the capability to achieve diversity gain for wireless networks. Therefore, the possibility of and mechanism for achieving greater utility for the CRN are studied when cooperative diversity is incorporated. To accomplish this, a resource allocation (RA) model is developed and analyzed for the heterogeneous, cooperative CRN. In the model, during cooperation, a best relay is selected to assist the secondary users (SUs) that have poor channel conditions. Overall, the cooperation makes it feasible for virtually all the SUs to improve their transmission rates while still causing minimal harm to the PUs. The results show a marked improvement in the RA performance of the CRN when cooperation is employed in contrast to when the CRN operates only by direct communication.

**Keywords:** Cognitive Radio Networks; Cooperative Diversity; Heterogeneous Networks; Resource Allocation.

## 1. Introduction

The measurements in various locations on the usage of the allotted spectrum spaces by the networks have shown a rather high level of inefficiency in spectrum usage [1, 2]. An outstanding attempt to resolve the issue are cognitive radio networks (CRN) [3]. Generally, with the CRN, unlicensed cognitive or secondary users (SUs) are made to access and utilize the same spectrum space that has been preallocated to primary users (PUs) of the spectrum, provided certain preconditions. In order to achieve an optimal productivity level for the resource allocation of the CRN, it is best to allocate low data rates to subchannels where the interference gains to PUs are quite high. This is understandable; as allocating high data rates to such subchannels would imply high transmit power by the SUs and high interference to the PUs because of the high interference gain. This smart move by the allocating algorithms of the SUs increases the throughput of the CRN. However, the achieved throughput is still very limited. In this work, as a result of recruiting cooperative diversity, a significant increase in reliability and capability of the system is realized.

## 2. Related Work

The concept of RA in CRN is no longer entirely new. Several research projects have been undertaken in this regard, and a review of relevant ones is performed in this section. Resource allocation in CRN actually deals with devising and describing mechanisms for assigning resources (frequency spectrum, transmit power, bandwidth, time slot, modulation scheme, etc) fairly and optimally to all users so that the highest possible productivity level is achieved.

A number of RA problems for underlay CRN have been identified, and attempts at solving them (both optimally and sub-optimally) have been investigated. In [4] Using game theory approach, the strong duality in convex optimization and the primal decomposition method, a low complexity semi-distributed algorithm was proposed for spectrum sharing and power allocation for MIMO-MB-CRNs. Other similar works that have developed RA models for underlay CRN can be found in [5-13]. References [14-15] have all developed models that describe possible cooperation between SUs in a CRN to help achieve a higher utility level. In [14], relays using decode-and-forward protocol are made to assist the SUs of the CRN. A similar model is developed in [15], where a decode-and-forward cooperative relay network is used to assist the SUs, thereby improving throughput.

As a means of addressing some of the limitations of the underlay and overlay arrangements, recent attempts at introducing user cooperation into RA in CRN have been made. A number of studies [14]–[17] have all developed models that describe possible cooperation between SUs in a CRN to help achieve a higher utility level. In two studies [14] relays using decode-and-forward protocol are made to assist the SUs of the CRN. For the optimization problem that has been developed to be solvable, the subchannels are first assigned to the SUs on the basis of their channel gains and possible interference to PUs. Thereafter, power is allocated to each subchannel.

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A similar model is developed in [10] where a decode-and-forward cooperative relay network is used to assist the SUs, thereby improving throughput. The nonconvex optimization problem that was developed is solved by first dualizing, then decomposing into relay assignment and power allocation. A primary decomposition method is also used in the work of Du et al, [16] after the power allocation problem in the developed model has been formulated. The sum rate of both PUs and SUs is jointly maximized in [17] while the SUs cooperate to transmit their signals. To achieve a result close to optimal, subchannels are first allocated to the SUs; thereafter, power is assigned to each SU and PU iteratively.

While the above reviewed works have incorporated some kind of cooperation, this work differs from them all in that the cooperative diversity approach developed in this work is targeted directly at addressing the problem of PU interference. Thus, the interference problem is first taken care of by the cooperation model even before the RA to SUs is performed. More specifically, in this work, through SU cooperation, the impact of the interference to PUs is mitigated, thus achieving greater throughput for the heterogeneous CRN. The heterogeneity in the CRN has been approached from 2 perspectives. Firstly, the channels are assumed to be heterogeneous, meaning that the available channels for the CRN do not all have the same characteristics. To capture the differing effects of channel heterogeneity, the network has been developed using an orthogonal frequency division multiple access (OFDMA) platform. With the OFDMA, the system can dynamically and optimally use different portions of the spectrum for different users at the same time. Secondly, the SUs in the network are heterogeneous. Users in each category are then serviced on the basis of their priority and/or their varying demands. During cooperation, the selection scheme used is the single-best-relay selection scheme used alongside the store-and-forward cooperative diversity technique. With this scheme, a best relay among the SUs is selected as the cooperator, which, during cooperation receives data from the source user and transmits to the destination. Overall, the heterogeneous cooperative CRN model, as developed and studied, reveals that much greater productivity is achievable by the CRN when its users cooperate. The contributions of this work are as follows:

- Investigating the use of cooperative diversity as a means of mitigating the limiting effects of interference to PUs in the RA problem of the CRN.
- Developing and analyzing methods for obtaining solutions to the RA problem in heterogeneous, cooperative CRN.

Section 3 describes the system model, Section 4 deals with the problem formulation and optimal solutions, Section 5 presents the heuristic developed to reduce the computational complexity, Section 6 presents the results and finally, Section 7 concludes the paper.

## 3. System Model

The CRN model consists of $K$ heterogeneous SUs and $L$ PUs, all within the coverage range of the secondary user base station (SUBS). $N$ OFDMA subchannels are available for the SUs. The $K$ heterogeneous SUs have different demands and priorities. These SUs are thus categorized as $K_1$ : SUs with minimum rate guarantee, and $(K-K_1)$: SUs with best effort service. All subchannels are in slow fading. Fig. 1. shows the network when cooperation is employed. The SU that needs cooperation, as it intends to communicate with a destination terminal (D), is referred to as the source secondary user (SSU). This SSU has a potentially high interference channel gain to the PU on the direct link and would therefore either not have been allocated subchannels at all or would have been given only a few subchannels to transmit at low data rates if direct communication alone had been considered. To help mitigate this limitation, the SSU selects a cooperating secondary user (CSU) with good channel quality and poor interference channel gain to the PU. The combined channel condition of the SSU and the CSU is obtained as follows:

Denote $H^s_{k,n}$ as the channel gain between the SSU and the $k$-th SU, employed as the CSU, at the $n$-th subchannel and $H^r_{k,n}$ as the channel gain between the CSU and the destination terminal D over the $n$-th subchannel. The SSU transmits signals to the $k$-th relay on the $n$-th subchannel with power $P^s_{k,n}$ in the first slot, while the $k$-th relay (CSU) transmits signals to D on the $n$-th subchannel with power $P^r_{k,n}$ in the second slot.

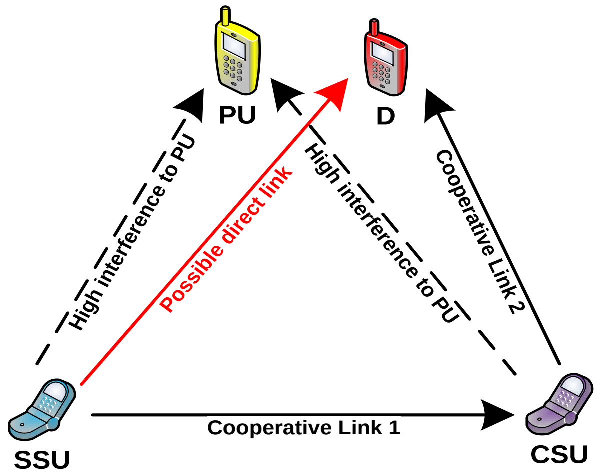


Fig. 1 System Model

Thus, the data rate of each slot is given as:

$$c^s_{k,n} = \log\left(1+\frac{P^s_{k,n}\left|H^s_{k,n}\right|^2}{\sigma_r^2+\sum_{l=1}^{L} J^l_{k,n}}\right), c^r_{k,n} = \log\left(1+\frac{P^r_{k,n}\left|H^r_{k,n}\right|^2}{\sigma^2+\sum_{l=1}^{L} J^l_{n}}\right)$$

where $\sigma_r^2$ and $\sigma^2$ are the noise powers at the $k$-th relay (CSU) and D respectively and the interference to the $k$-th relay and that to D on the $n$-th subchannel by the $l$-th PU's signal are denoted by $J_{k,n}^l$ and $J_n^l$.

The cooperative data rate is limited by the minimum of the two hops:

$$c_{k,n,C} = \min\left(c_{k,n}^s, c_{k,n}^r\right) \quad (2)$$

$c_{k,n,D}$ denotes the data rate over direct path from the SU to D. This data rate $c$ for each subchannel is dependent on the modulation scheme. In this work, four modulation schemes, which are binary phase shift keying (BPSK), 4-quadrature amplitude modulation (QAM), 16-QAM and 64-QAM, are considered. The modulation schemes transmit $c = 1, 2, 4$ and 6 bits per OFDMA symbol respectively. For a given BER $\rho$, the minimum power for BPSK modulation is given as $P(c,\rho) = N_\phi\left[c \times erfc^{-1}(2\rho)\right]^2$ (where $c = 1$ ), while for the M-ary QAM, the minimum power is given as $P(c,\rho) = \frac{2(2^c-1)N_\phi}{3}\left[erfc^{-1}\left(\frac{c\rho\sqrt{2^c}}{2\left(\sqrt{2^c}-1\right)}\right)\right]^2$ where $N_\phi$ is the single-sided noise power spectral density. The minimum power $P_{k,n}\left(c_{k,n},\rho\right)$ required at the $k$-th SU over the nth subchannel to transmit $c_{k,n}$ bits is obtained by dividing the power of that user $k$ on the $n$-th subchannel by the channel gain between the SUBS and the user $k$ over that subchannel n. This is thus given as:

$$P_{k,n}\left(c_{k,n},\rho\right) = \frac{P\left(c_{k,n},\rho\right)}{H_{k,n}^c} \quad (3)$$

## 4. Problem Formulation

Let the minimum data rate assigned to user $k$ in category one be $R_k$ and the normalized proportional fairness factor for each SU in category two be $\gamma_k$ with data rate $R_i$ indicating the rate for the element $i$. The total power on the $n$-th subchannel is represented as $\Phi_n = \sum_{k=1}^{K} P_{k,n}$ with $P_{k,n}$ being the transmit power of user $k$ over the nth subchannel. Also let the interference power gain matrix between the SUBS and the available PU be represented as $\mathbf{H}^p \in \mathbb{R}^{L\times N}$. The vector $\mathbf{H}_{l,n}^p$ therefore denotes the subchannel interference power gain between the SUBS and PU $l$ over subchannel $n$. The maximum permissible level of interference to the $l$-th PU from all the transmitting SUs is represented as $\varepsilon_l$ while $P_{\max}$ denotes the maximum transmit power at the SUBS. Also let $X_{k,n,D}$ be a binary variable employed to limit each subchannel to direct or cooperative communication. The resource allocation problem for the heterogeneous cooperative CRN is formulated as:

$$z = \max \sum_{n=1}^{N}\left(\sum_{k=1}^{K_1}\left[X_{k,n,D}c_{k,n,D} + \left(1-X_{k,n,D}\right)c_{k,n,C}\right] + \sum_{k=K_1+1}^{K}\left[X_{k,n,D}c_{k,n,D} + \left(1-X_{k,n,D}\right)c_{k,n,C}\right]\right) \quad (4)$$

subject to

$$\sum_{n=1}^{N}\left(c_{k,n,D} + c_{k,n,C}\right) \geq R_k; k = 1,2,\ldots,K_1 \quad (5)$$

$$\frac{R_k}{\sum_{i=K_1+1}^{K} R_i} = \gamma_k; k = K_1+1, K_1+2,\ldots,K \quad (6)$$

$$\sum_{n=1}^{N}\left(\sum_{k=1}^{K}\left[X_{k,n,D}P_{k,n,D} + \left(1-X_{k,n,D}\right)P_{k,n,C}\right]\right) \leq P_{\max} \quad (7)$$

$$\sum_{n=1}^{N}\Phi_n \mathbf{H}_{l,n,D}^p \leq \varepsilon_l; l = 1,2,\ldots,L \quad (8)$$

$$\sum_{n=1}^{N}\Phi_n \mathbf{H}_{l,n,C}^p \leq \varepsilon_l; l = 1,2,\ldots,L \quad (9)$$

$$\begin{cases} c_{k,n,D} = 0 & \text{if } c_{k',n,D} \neq 0, \\ c_{k,n,C} = 0 & \text{if } c_{k',n,C} \neq 0, \end{cases} \forall k' \neq k; k = 1,2,\ldots,K \quad (10)$$

$$X_{k,n,D} \in \{0,1\}, X_{k,n,D} = \begin{cases} 1 & \text{if } c_{k,n,D} \neq 0 \\ 0 & \text{otherwise} \end{cases} \quad (11)$$

The equation in constraint (6) can be equally expressed as:

$$R_k = \gamma_k \times \sum_{i=K_1+1}^{K} R_i$$

Representing $\gamma_k \times \sum_{i=K_1+1}^{K} R_i$ by $\tilde{\gamma}_k$, Equation (6) becomes:

$$R_{K_1+1} : R_{K_1+2} : \ldots : R_K = \tilde{\gamma}_{K_1+1} : \tilde{\gamma}_{K_1+1} : \ldots : \tilde{\gamma}_K \quad (12)$$

The above formulation of the RA problem is not a linear programming problem. However, the problem can be reformulated as an integer linear programming (ILP) problem. The reformulated problem can easily be solved by using any of the classical optimization techniques. The branch-and-bound (BnB) approach is used in this work. The reformulation is achieved as described below:

Define $\mathbf{x}_I$ as the bit allocation vector for all subchannels assigned to all users in category one and also define $\mathbf{x}_{II}$ as the bit allocation vector for all subchannels assigned to all users in category two. $\mathbf{x}_I$ and $\mathbf{x}_{II}$ are given as:

$$\mathbf{x}_I = \left[\left(\mathbf{x}_{I,N}^1\right)^T \quad \left(\mathbf{x}_{I,N}^2\right)^T \quad \ldots \quad \left(\mathbf{x}_{I,N}^N\right)^T\right]^T \in \{0,1\}^{NK_1C\times 1} \quad (13)$$

$$\mathbf{x}_{II} = \left[\left(\mathbf{x}_{II,N}^1\right)^T \quad \left(\mathbf{x}_{II,N}^2\right)^T \quad \ldots \quad \left(\mathbf{x}_{II,N}^N\right)^T\right]^T \in \{0,1\}^{N(K-K_1)C\times 1} \quad (14)$$

where $\mathbf{x}_{I,N}^n = \left[x_{I,1,n}^T \quad x_{I,2,n}^T \quad \ldots \quad x_{I,K,n}^T\right]^T \in \{0,1\}^{KC\times 1}$ indicates that subchannel $n$ has been assigned to a category one SU with

$\mathbf{x}_{I,k,n} = \begin{bmatrix} x_{k,n,1} & x_{k,n,2} & \cdots & x_{k,n,M} \end{bmatrix}^T \in \{0,1\}^{C\times 1}$ ; $n = 1,\ldots,N$ ; $k = 1,\ldots,K$ ; $M$ indicates the overall number of modulation schemes being employed (for this work, $M = 4$ ). The implication is that $\mathbf{x}_{I,k,n} = \begin{bmatrix} x_{k,n,1} & x_{k,n,2} & x_{k,n,3} & x_{k,n,4} \end{bmatrix}^T$ . Similar explanations apply to $\mathbf{x}_{II}$ . The combined bit allocation vector $\mathbf{x} = \mathbf{x}_I + \mathbf{x}_{II}$ . Because of the mutually exclusive constraint, $\mathbf{x}_{I,N}^n$ and $\mathbf{x}_{II,N}^n$ can be any of the vectors $\left\{\begin{bmatrix}0 & 0 & \ldots & 0\end{bmatrix}^T, \begin{bmatrix}1 & 0 & \ldots & 0\end{bmatrix}^T, \begin{bmatrix}0 & 1 & \ldots & 0\end{bmatrix}^T, \ldots, \begin{bmatrix}0 & 0 & \ldots & 1\end{bmatrix}^T\right\}$

Hence, only one component in $\mathbf{x}_{I,N}^n$ is 1, while the other components are all 0s. If $x_{k,n,c}$ is 1, it means that subchannel $n$ has been assigned to user $k$ to transmit $c$ bits per symbol. If $\mathbf{x}_{I,N}^n$ has all its components as 0s, subchannel $n$ is not being assigned to any user. For the two user categories, define the modulation order vectors $\mathbf{b}_I$ and $\mathbf{b}_{II}$ as:

$$\mathbf{b}_I = \left[ \left(b_{I,N}^1\right)^T \quad \left(b_{I,N}^2\right)^T \quad \cdots \quad \left(b_{I,N}^N\right)^T \right]^T \in \mathbb{R}^{NK_1C\times 1} \quad (15)$$

$$\mathbf{b}_{II} = \left[ \left(b_{II,N}^1\right)^T \quad \left(b_{II,N}^2\right)^T \quad \cdots \quad \left(b_{II,N}^N\right)^T \right]^T \in \mathbb{R}^{N(K-K_1)C\times 1} \quad (16)$$

where $\mathbf{b}_{I,N}^n = \begin{bmatrix} b_{I,1,n}^T & b_{I,2,n}^T & \ldots & b_{I,K,n}^T \end{bmatrix}^T$ , $\mathbf{b}_{I,k,n}^n = \begin{bmatrix} b_{k,n,1}^T & b_{k,n,2}^T & \ldots & b_{k,n,C}^T \end{bmatrix}^T$ . Similar explanations also apply to $\mathbf{b}_{II}$ . Having considered only four modulation schemes, $\mathbf{b}_{1,k,n} = \begin{bmatrix}1 & 2 & 3 & 4\end{bmatrix}^T$ (the same applies to $\mathbf{b}_{II,N}^n$ ). For the two categories of SUs, data rate matrices $\mathbf{B}_i \in \mathbb{R}^{K_1\times NK_1C}$ and $\mathbf{B}_j \in \mathbb{R}^{(K-K_1)\times N(K-K_1)C}$ are defined respectively as:

$$\mathbf{B}_i = \begin{bmatrix} b_1 & b_1 & \ldots & b_1 \\ b_2 & b_2 & \ldots & b_2 \\ \vdots & \vdots & \ddots & \vdots \\ b_{K_1} & b_{K_1} & \ldots & b_{K_1} \end{bmatrix}, \left\{ \begin{matrix} b_1 = \begin{bmatrix} b^T & 0_C^T & \cdots & 0_C^T \end{bmatrix} \in \mathbb{R}^{1\times K_1C} \\ b_2 = \begin{bmatrix} 0_C^T & b^T & \cdots & 0_C^T \end{bmatrix} \in \mathbb{R}^{1\times K_1C} \\ \vdots \quad \vdots \quad \ddots \quad \vdots \\ b_{K_1} = \begin{bmatrix} 0_C^T & 0_C^T & \cdots & b^T \end{bmatrix} \in \mathbb{R}^{1\times K_1C} \end{matrix} \right\}$$

(17)

$$\mathbf{B}_j = \begin{bmatrix} b_{K_1+1} & b_{K_1+1} & \ldots & b_{K_1+1} \\ b_{K_1+2} & b_{K_1+1} & \ldots & b_{K_1+1} \\ \vdots & \vdots & \ddots & \vdots \\ b_K & b_K & \ldots & b_K \end{bmatrix}, \left\{ \begin{matrix} b_{K_1+1} = \begin{bmatrix} b^T & 0_C^T & \cdots & 0_C^T \end{bmatrix} \in \mathbb{R}^{1\times (K-K_1)C} \\ b_{K_1+2} = \begin{bmatrix} 0_C^T & b^T & \cdots & 0_C^T \end{bmatrix} \in \mathbb{R}^{1\times (K-K_1)C} \\ \vdots \quad \vdots \quad \ddots \quad \vdots \\ b_K = \begin{bmatrix} 0_C^T & 0_C^T & \cdots & b^T \end{bmatrix} \in \mathbb{R}^{1\times (K-K_1)C} \end{matrix} \right\}$$

(18)

Define $\mathbf{R}_k \triangleq \begin{bmatrix} R_1 & R_2 & \ldots & R_{K_1} \end{bmatrix}^T$ and $\tilde{\gamma}_k \triangleq \begin{bmatrix} \tilde{\gamma}_{K_1+1} & \tilde{\gamma}_{K_1+2} & \ldots & \tilde{\gamma}_K \end{bmatrix}^T$ , the constraint of Equation (5) can be written as $\mathbf{B}_i\mathbf{x}_I \geq \mathbf{R}_k$ , while the data rate constraint for category two SU can be written as $\mathbf{B}_j\mathbf{x}_{II} = \tilde{\gamma}_k$. A power transmission vector $\mathbf{p}$ is defined as:

$$\mathbf{p} = \left[ \left(\mathbf{p}_N^1\right)^T \quad \left(\mathbf{p}_N^2\right)^T \quad \cdots \quad \left(\mathbf{p}_N^N\right)^T \right]^T \in \mathbb{R}^{NKC\times 1} \quad (19)$$

where $\mathbf{p}_N^n = \begin{bmatrix} \mathbf{p}_{1,n}^T & \mathbf{p}_{2,n}^T & \cdots & \mathbf{p}_{K,n}^T \end{bmatrix}^T$ , $\mathbf{p}_{k,n} = \begin{bmatrix} p_{k,n,1} & p_{k,n,2} & \cdots & p_{k,n,C} \end{bmatrix}^T$ ; $p_{k,n,c}$ is the power required to transmit $c$ bits over subchannel $n$ for user $k$. Equation (7) can be written as $\mathbf{p}^T\mathbf{x} \leq P_{\max}$ . The transmit power is the sum of the powers used for both direct and cooperation transmission, $\mathbf{p} = \mathbf{p}_D + \mathbf{p}_C$ , where $\mathbf{p}_D$ and $\mathbf{p}_C$ are the transmit power vectors during direct and cooperation transmission respectively. The power constraint therefore becomes $\left(\mathbf{p}_D + \mathbf{p}_C\right)^T \mathbf{x} \leq P_{\max}$ . To write Equation (8), the interference power constraint in terms of the bit allocation vector $\mathbf{x}$ , define a matrix $\mathbf{A} \in \{0,1\}^{N\times NKC}$ as:

$$\mathbf{A} = \begin{bmatrix} 1_{KC}^T & 0_{KC}^T & \cdots & 0_{KC}^T \\ 0_{KC}^T & 1_{KC}^T & \cdots & 0_{KC}^T \\ \vdots & \vdots & \ddots & \vdots \\ 0_{KC}^T & 0_{KC}^T & \cdots & 1_{KC}^T \end{bmatrix}, 1_{KC} = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} \in \{1\}^{KC\times 1}, 0_{KC} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \in \{0\}^{KC\times 1}$$

(20)

Let $\mathbf{p} \circ \mathbf{x}$ be the Schur-Hadamard (or entry-wise) product of $\mathbf{p}$ and $\mathbf{x}$ . By defining $\varepsilon_l \triangleq \begin{bmatrix} \varepsilon_1 & \varepsilon_2 & \ldots & \varepsilon_L \end{bmatrix}^T \in \mathbb{R}^{L\times 1}$ , Equation (8) can then be written as:

$$\left[\mathbf{H}_{l.n,D}^p \left(\mathbf{A}\left(\mathbf{P}_D \circ \mathbf{x}\right)\right)\right] \leq \varepsilon_l \quad (21)$$

Likewise, the constraint in Equation (9) can be written as:

$$\left[\mathbf{H}_{l.n,C}^p \left(\mathbf{A}\left(\mathbf{P}_C \circ \mathbf{x}\right)\right)\right] \leq \varepsilon_l \quad (22)$$

Thus, the resource allocation problem for the modeled heterogeneous cognitive CRN described in Equations (4) - (11) can be described in the ILP form as:

$$z^* = \max_{\mathbf{x}} \left[ \left(\mathbf{b}_I\right)^T \mathbf{x}_I + \left(\mathbf{b}_{II}\right)^T \mathbf{x}_{II} \right] \quad (23)$$

subject to $\mathbf{B}_i\mathbf{x}_I \geq \mathbf{R}_k ; k = 1,2,\ldots,K_1$ (24)

$$\mathbf{B}_j\mathbf{x}_{II} = \tilde{\gamma}_k ; k = K_1+1, K_1+2,\ldots,K \quad (25)$$

$$\left(\mathbf{p}_D + \mathbf{p}_C\right)^T \mathbf{x} \leq P_{\max} \quad (26)$$

$$\left[\mathbf{H}_{l.n,D}^p \left(\mathbf{A}\left(\mathbf{P}_D \circ \mathbf{x}\right)\right)\right] \leq \varepsilon_l \quad (27)$$

$$\left[\mathbf{H}_{l.n,C}^p \left(\mathbf{A}\left(\mathbf{P}_C \circ \mathbf{x}\right)\right)\right] \leq \varepsilon_l \quad (28)$$

$$\mathbf{0}_N \leq \mathbf{A}\mathbf{x} \leq \mathbf{1}_N \quad (29)$$

$$\mathbf{x}_I, \mathbf{x}_{II}, \mathbf{x} \in \{0,1\} \quad (30)$$

The formulation above is an ILP problem in which, in this work, the BnB approach has been employed to obtain solutions. BnB optimization is a very useful and well-developed technique for solving such problems.

## 5. Iterative based Heuristic

In this section, a fast, iterative-based heuristic is developed to solve the ILP problem. The algorithm involves two steps:

- subchannel allocation

- iterative bit and power allocation.

Subchannel Allocation

In carrying out the subchannel allocation for the different categories of SUs, the constraint $\mathbf{x} \in [0,1]$ is integer-relaxed such that the constraint becomes:

$$0 \le \mathbf{x} \le 1 \quad (31)$$

By solving this integer-relaxed formulation at the first iteration, the values of $\mathbf{x}$ are obtained. The data rate for the *k*-th SU at the *n*-th subchannel becomes $\left(\mathbf{b}_{k,n}^T \mathbf{x}_{k,n}\right)$. The subchannel $n$ is only allocated to user k after ascertaining that $\left(\mathbf{b}_{k,n}^T \mathbf{x}_{k,n}\right) \ge \left(\mathbf{b}_{m,n}^T \mathbf{x}_{m,n}\right) \quad \forall m \ne k$. Clearly

then, each subchannel is allocated to the SU that has the highest achievable data rate over that subchannel. It is important to realize too that once the suchannels have been allocated to the different SUs using the above criterion at the first iteration, the dimension of $\mathbf{x}$ reduces from its initial value of $\mathbf{x} \in [0,1]^{KNC\times 1}$ to the smaller value of $\mathbf{x} \in [0,1]^{NC\times 1}$.

## 5.1 Binary SAR Architecture

Once the subchannels have been assigned to the SUs, it remains to determine how many bits and what power can be associated with each subchannel. This is performed in an iterative manner. The optimization process occurs in a number of iterations, say *y*. In general, the following optimization problem has to be solved at the *y*-th iteration step:

$$\max_{\mathbf{x}^y} \left[ \left(\mathbf{b}_I^y\right)^T \mathbf{x}_I^y + \left(\mathbf{b}_{II}^y\right)^T \mathbf{x}_{II}^y \right] \quad (32)$$

subject to $\mathbf{B}_i \mathbf{x}_I^y \ge \left[\mathbf{R}_k - \mathbf{f}^{(y-1)}\right]^+, k = 1,2,\ldots,K_1 \quad (33)$

$$\mathbf{B}_j \mathbf{x}_{II}^y = \left[\tilde{\gamma}_k - \mathbf{g}^{(y-1)}\right]^+, k = K_1+1, K_1+2,\ldots,K \quad (34)$$

$$\left(\mathbf{p}^{(y-1)}\right)^T \mathbf{x}^y \le P_{\max} - \left\|\mathbf{u}^{(y-1)}\right\|_1 \quad (35)$$

$$\mathbf{H}^p \left[\mathbf{A}\left(\mathbf{p}^{(y-1)} \odot \mathbf{x}^y\right)\right] \le \varepsilon_l - \mathbf{H}^p \mathbf{u}^{(y-1)} \quad (36)$$

$$\mathbf{0}_N \le \mathbf{A}\mathbf{x}^y \le \mathbf{1}_N \quad (37)$$

$$\mathbf{0}_{KNC} \le \mathbf{x}^y \le \mathbf{1}_{KNC} \quad (38)$$

where $\mathbf{f}^{(y-1)}$ and $\mathbf{g}^{(y-1)}$ are the allocated bits for category one and category two users at the *y*-th iteration respectively, and $\mathbf{u}^{(y-1)}$ is the allocated power at the *y*-th iteration. Here, a detailed explanation on the iteration process is given. Recall that the bit allocation to the *n*-th subchannel assigned to a category one SU, $\mathbf{b}_{I,n} = \left[b_{I,n}^T \ \ldots \ b_{K_1,n}^T\right]^T$ is a vector of size $K_1 C \times 1$ with possible entries 1, 2, 4 and 6. Assume that during the subchannel allocation carried out in the last subsection, the first subchannel has been allocated to the second user, which happens to be a category one SU. Then, $\mathbf{b}_{I,1} = [0 \ 0 \ 0 \ 0,1 \ 2 \ 4 \ 6,0 \ 0 \ 0 \ 0,0 \ 0 \ 0 \ 0]$ for users in category one (assuming there are four users). If it had been the third subchannel that was allocated to the first user, which happens to be a category two SU, then $\mathbf{b}_{II,3} = [1 \ 2 \ 4 \ 6,0 \ 0 \ 0 \ 0,0 \ 0 \ 0 \ 0,0 \ 0 \ 0 \ 0]$ (assuming there are also four users in this category) and so on. Once this has been done and certain elements of $\mathbf{b}_I$ and $\mathbf{b}_{II}$ are zeros according to the subchannel allocation, the vectors $\mathbf{b}_I$ and $\mathbf{b}_{II}$ are renamed $\mathbf{b}_I^1$ and $\mathbf{b}_{II}^1$ respectively. Consequently, at the first iteration (i.e. when $y = 1$), the following optimization problem is solved:

$$\max_{\mathbf{x}^1} \left[ \left(\mathbf{b}_I^1\right)^T \mathbf{x}_I^1 + \left(\mathbf{b}_{II}^1\right)^T \mathbf{x}_{II}^1 \right] \quad (39)$$

subject to $\mathbf{B}_i \mathbf{x}_I^1 \ge \mathbf{R}_k, k = 1,2,\ldots,K_1 \quad (40)$

$$\mathbf{B}_j \mathbf{x}_{II}^1 = \tilde{\gamma}_k, k = K_1+1, K_1+2,\ldots,K \quad (41)$$

$$\mathbf{p}^T \mathbf{x}^1 \le P_{\max} \quad (42)$$

$$\mathbf{H}_{l,n,D}^p \left[\mathbf{A}\left(\mathbf{p}_D \odot \mathbf{x}^1\right)\right] \le \varepsilon_l \quad (43)$$

$$\mathbf{H}_{l,n,C}^p \left[\mathbf{A}\left(\mathbf{p}_C \odot \mathbf{x}^1\right)\right] \le \varepsilon_l \quad (44)$$

$$\mathbf{0}_N \le \mathbf{A}\mathbf{x}^1 \le \mathbf{1}_N \quad (45)$$

$$\mathbf{0}_{KNC,1} \le \mathbf{x}^1 \le \mathbf{1}_{KNC,1} \quad (46)$$

The rates $\mathbf{B}_i \mathbf{x}_I^1$ and $\mathbf{B}_i \mathbf{x}_{II}^1$ and power $\mathbf{p}^T \mathbf{x}^1$ obtained at the first iteration are passed on as $\mathbf{f}^1, \mathbf{g}^1$ and $\mathbf{u}^{(1)}$ respectively for the second iteration. Vector $\mathbf{x}^1$ is used along with the power vector $\mathbf{p}$ to determine the initial modulation scheme for each SU at various subchannels. The total power allocated to the first subchannel can then be calculated as $\left(p_{k,n}^T x_{k,n}^1\right)$. To generalize, if the *n*-th subchannel is allocated to the *k*-th SU, the total power allocated to it is calculated as $\left(p_{k,n}^T x_{k,n}^1\right)$. The modulation scheme $q$ (with bits $c_q$) that can be employed without exceeding the power $\left(p_{k,n}^T x_{k,n}^1\right)$ can be obtained as:

$$q = \arg\max_q \left\{ q \in [0,1,2,3,4] : p_{k,n,q} \le p_{k,n}^T x_{k,n}^1 \right\} \quad (47)$$

The interference leakage to PU will still be less than $\varepsilon$. As a result, it is most likely that there will be some residual power available for use. Hence $y = 2$ becomes feasible. Since the first subchannel has been allocated to the second user, which happens to be in category one, to transmit 2 bits, then, $\mathbf{b}_{I,2,1}$ can be modified as $\mathbf{b}_{I,2,1} = [0 \ 0 \ 2 \ 4]^T$. To have allocated 2 bits to this subchannel, the power $\mathbf{p}_{2,1,2}$ must have been used. With the realization of excess power available for use, the allocation might then be upgraded to, say, a 16-QAM (to transmit 2 more bits) or 64-QAM. For this to take place would require additional power of $\left(p_{2,1,3} - p_{2,1,2}\right)$ (for 16-QAM) or $\left(p_{2,1,4} - p_{2,1,2}\right)$ (for 64-QAM) respectively.

Hence, the new power vector at the second iteration $\mathbf{p}_{2,1}^{1} = \left[ p_{2,1,1} \quad p_{2,1,2} \quad \left(p_{2,1,3} - p_{2,1,2}\right) \quad \left(p_{2,1,4} - p_{2,1,2}\right)\right]^{T}$ . If $u_n^1$ denotes the power that was allocated to the $n$-th subchannel in the first iteration, then $\mathbf{u}^1 \square \left[ u_1^1 \quad \dots \quad u_N^1 \right]^T$ It therefore implies that $P_{\max} - \sum_{n=1}^{N} u_n^1$ is the residual power available for the second iteration step. This total power is given as $\upsilon_n^2 = u_n^1 + \left(\mathbf{p}_{k,n}^1\right)^T \mathbf{x}_{k,n}^2$ . This new power is used to decide about the modulation scheme $q$ of the $n$-th subchannel should be upgraded to:

$$q = \arg\max_{q} \left\{ q \in [0,1,2,3,4] : p_{k,n,q} \le \upsilon_{k,n}^2 \right\} \quad (48)$$

Similarly, the interference to PUs as a result of the power allocated in the first iteration step is given as $\mathbf{H}^p\mathbf{u}^1$ . The remaining interference permissible must be less than $\left(\varepsilon_l - \mathbf{H}^p\mathbf{u}^1\right)$ for the second iteration. Since, at this second iteration, $f_k^1$ already becomes the data rate allocated to the $k$-th SU in category one during the first iteration and $g_k^1$ becomes the data rate already allocated to the $k$-th SU in category two during the first iteration, $\mathbf{f}^1$ and $\mathbf{g}^1$ are defined as $\mathbf{f}^1 \square \left[ f_1^1 \quad \dots \quad f_1^k \right]^T$ and $\mathbf{g}^1 \square \left[ g_1^1 \quad \dots \quad g_1^k \right]^T$ respectively. Hence, the data rate requirement at the second iteration for category one users would be $\left(\mathbf{R}_k - \mathbf{f}^1\right)$, while the available data rate for category two users at the second iteration would be $\left(\tilde{\gamma}_k - \mathbf{g}^1\right)$. The constraints on data rate then become $\mathbf{B}_i\mathbf{x}_I^2 \ge \left[\mathbf{R}_k - \mathbf{f}^1\right]^+$ for category one users and $\mathbf{B}_j\mathbf{x}_{II}^2 = \left[\tilde{\gamma}_k - \mathbf{g}^1\right]^+$ for category two users. This whole iteration process is repeated continuously and only stopped when no further improvement can be achieved on the total achievable data rate for each user. The stopping criterion is thus given as:

$$\left[\left(\mathbf{b}_I^y\right)^T \mathbf{x}_I^y + \left(\mathbf{b}_{II}^y\right)^T \mathbf{x}_{II}^y\right] - \left[\left(\mathbf{b}_I^{y-1}\right)^T \mathbf{x}_I^{y-1} + \left(\mathbf{b}_{II}^{y-1}\right)^T \mathbf{x}_{II}^{y-1}\right] = \varsigma$$

(49)

where $\varsigma$ is a predetermined (very small) value. After the $y$-th iteration, the vectors $\mathbf{f}^{(y+1)}$ and $\mathbf{g}^{(y+1)}$ will contain the allocated bits for each subchannel assigned to category one and category two users respectively. The pseudo-code given in Table 1 summarizes both the subchannel allocation and the iterative bit and power allocation.

## 6. Simulation Results

In this section, the performance of the proposed algorithms is evaluated using appropriate simulations. A static system level simulation is done as a single cell, which contains 8 SUs, 4 PUs, with category one SUs $K_1 = 2$ , category two SUs $\left(K - K_1\right) = 2$ and SUs as possible cooperators from which the best relay (CSU) is selected = 4. The minimum data rate requirement for category one SUs is 64 bits/user. The area covers 2 km × 2 km and the SUs are uniformly distributed in the area. It is assumed here that 64 subchannels are available for secondary usage, meanwhile used by the PUs. The elements of the channel matrices follow a Rayleigh distribution and are independent of each other. The path-loss exponent is 4, and the standard deviation of shadowing is 6 dB. The noise power is $\sigma^2 = 10^{-12}$ W/Hz.

Table 1. Pseudo-code for the proposed iterative-based heuristic

| |
|---|
| • Pseudo-code for the subchannel allocation |
| • solve for x using Equations (23) - (29) and (31)<br>• set subchannel index $n = 0$<br>• $n \leftarrow n+1$<br>• if $\mathbf{b}_{k,n}^T \mathbf{x}_{k,n} \ge \mathbf{b}_{m,n}^T \mathbf{x}_{m,n}$ , $\forall m \ne k$<br>• $n$ -th subchannel is allocated to user $k$<br>• end if<br>• until $n < N+1$ |
| • Pseudo-code for the bit and power allocation |
| • set $n = 0, y = 0, \mathbf{u}^{(0)} = \mathbf{0}_N, \mathbf{p}^{(0)} = \mathbf{p}$<br>• $y \leftarrow y+1$<br>• set $\mathbf{f}^y = \mathbf{0}_K, \mathbf{g}^y = \mathbf{0}_K, \upsilon^y = \mathbf{0}_N$<br>• solve the problem (32) - (38)<br>• repeat<br>• $n < N+1$<br>• $\upsilon_n^y = u_n^{y-1} + \left(\mathbf{p}_{k,n}^{y-1}\right)^T \mathbf{x}_{k,n}^y$<br>• if $q = \arg\max_q \left\{ q \in [0,1,2,3,4] : p_{k,n,q} \le \upsilon_n^y \right\}$ then<br>• use modulation scheme $q$ on $n$-th subchannel<br>• set $u_{k,n}^y = p_{k,n,l}; f_k^y = f_k^y + c_q; g_k^y = g_k^y + c_q$<br>• set $p_{k,n,m}^y = p_{k,n,m} - p_{k,n,l}, \forall m > l$<br>• set $b_{k,n,m}^{y+1} = b_{k,n,l} - c_q, \forall m > l$<br>• set $b_{k,n,m}^{y+1} = 0, \forall m \le l$<br>• end if<br>• until $n < N+1$<br>• until no further improvement on total data rate (Equation (49))<br>• the vectors $\mathbf{f}^{y+1}$ and $\mathbf{g}^{y+1}$ contain the bits allocated for each subchannel in category one and two respectively<br>• the vector $\mathbf{u}^{y+1}$ contains the power allocated for each subchannel |

Figure 2 shows the interference channel gain patterns for the various PUs. At high interference gain, the subchannels are allocated low data rates to reduce the adverse effect of high power and/or interference gain on the PUs and vice versa. The combined interference to the PUs on subchannels 2, 3, 9, 57, 63, and 64 is lower than the combined interference on the other subchannels. On subchannels 14 to 27 and 39 to 52, the combined interference to PUs is quite high, and the subchannels have been allocated low data rates to transmit. This is the fundamental principle by which the bit allocation is performed to obtain optimal results on the overall throughput of the network.

Figure 3 gives the average data rate (bits per OFDMA symbol) for each category of SUs against the maximum interference power to the PUs for both direct communication and cooperative communication, where the SUBS maximum transmit power is at 20 dBm. From the results obtained, it is obvious that for the developed RA problem to have feasible solutions, the minimum rate constraint of the high priority category 1 users has to be met at all times. Importantly, the result shows that a marked improvement in performance of the network is achieved during cooperation, compared to when direct communication alone is used. The reason for this is the improved interference gain to PUs that the cooperative network achieves, which means that the subchannels can transmit at a higher rate than they would ordinarily have been allocated by direct communication. It is also worth noting that in Figure 3, the average data rate during cooperation eventually converges to nearly that of direct communication. This shows that as the permissible interference level to PUs increases, the need for and/or effect of cooperation diminishes. It would be better to transmit directly if the PUs are robust to the SUs' interference than to transmit using cooperation, as cooperation generally requires much more signaling overhead than direct communication.

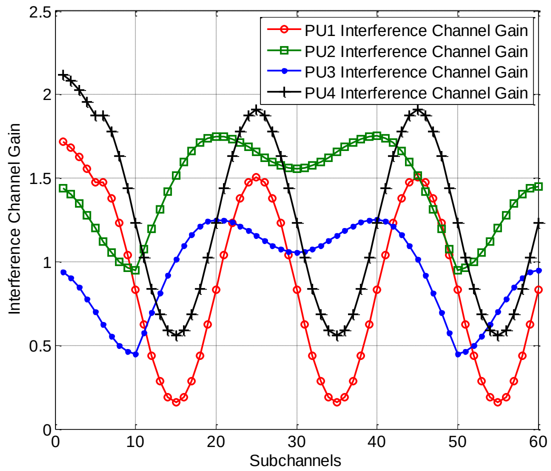


Fig. 2 Interference gain of PUs

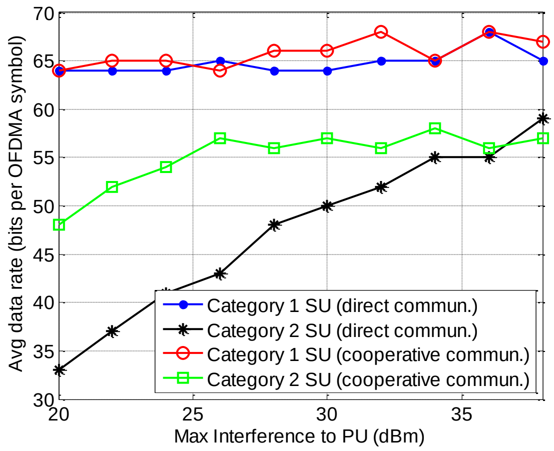


Fig. 3 Average data rate different categories of SUs

Figure 4 describes the average data rate performance for increasing SUBS power. The 2 categories of SUs are covered, and both direct and cooperative communications are considered. In Fig. 4, a maximum interference power to PUs of 25 dBm is used. At all times, the minimum rate guarantee of the category 1 SUs must be met for the problem to have feasible solutions. Again, as the SUBS power is increased, the average data rate improves, particularly for category 2 SUs. After a while though, no further improvement can be observed, irrespective of whether or not the SUBS power is increased. The reason for this is that the other constraints also come into play, thus making it impossible for the data rate to keep increasing indefinitely with increasing SUBS power. It is significant to note the improvement that cooperative communication achieves over direct communication. This improvement can be seen when the interference limit is at 25 dBm. These results clearly show that the network would rather transmit using direct communication when the SUBS power is limited so as to maximize the power use and reduce signalling overhead. At higher power, however, cooperative communication is preferred, as the overall capacity is remarkably better.

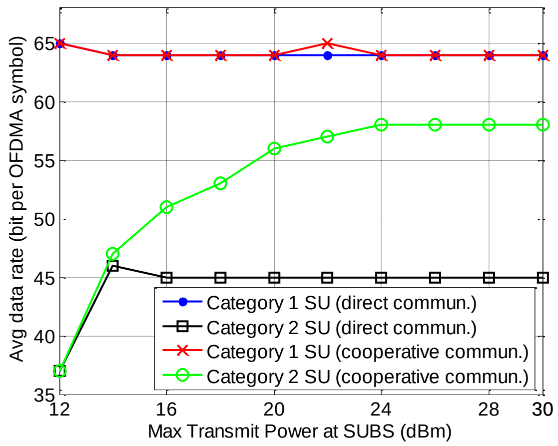


Fig. 4 Average data rate different categories of SUs

## 7. Conclusions

In CRN, RA models that can yield outstanding productivity even with very stringent constraints are critical. This work develops such a model whereby, in a heterogeneous CRN environment, cooperative diversity is used in mitigating the limiting effects of the interference to the PUs of the network. To make the model feasible and close to practical, only one single best relay is selected from the available ones as the cooperating relay. Also, cooperation is only used by users that have subchannels with a high interference gain to the PU. The problem that has been developed is first solved by a careful reformulation of the non-deterministic polynomial-time-hard problem into an ILP problem, and optimal solutions are obtained using the BnB method for solving ILP problems. To reduce computational complexity, an iterative-based heuristic is developed to solve the problem in a much reduced time frame. The results presented compare the average data rates and total data rates for the different categories of SUs when direct and cooperative communications are used. The optimality and computational complexity of the developed heuristic are compared with those obtained using ILP as well. The improvement in the performance of the network when cooperation is used is quite remarkable, as the results have shown.

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